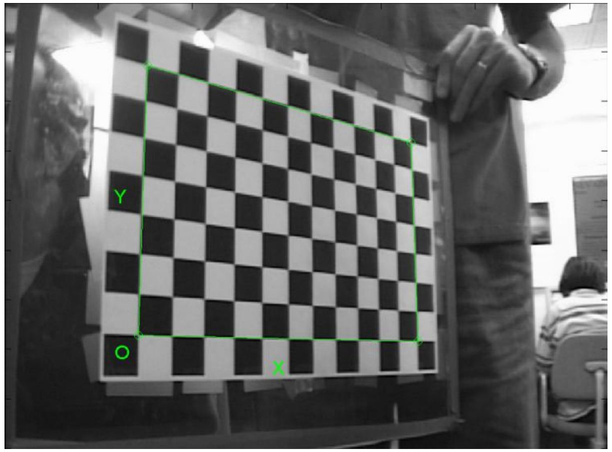
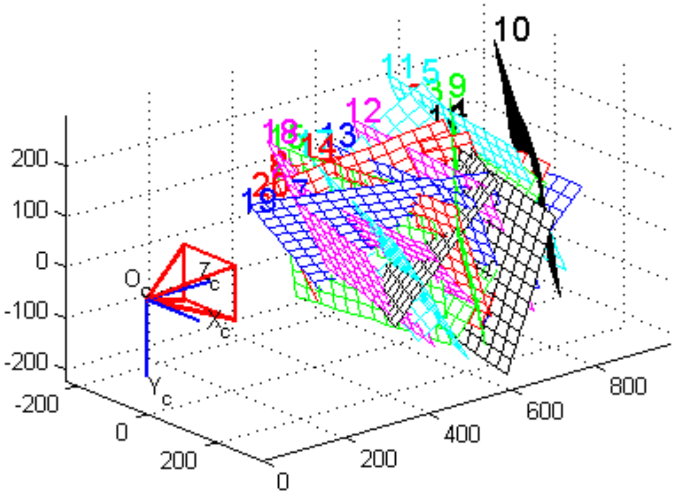
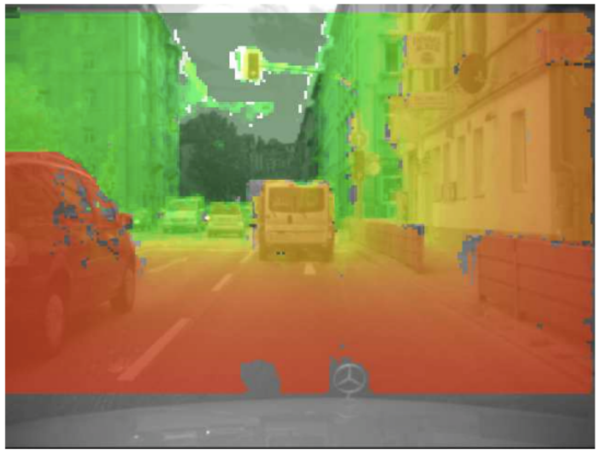
Camera Calibration & Depth Estimation

Kuan-Wen Chen
2018/3/22







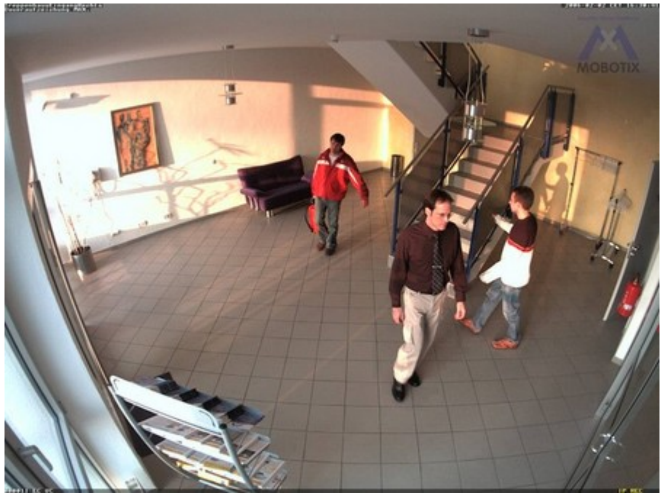
Camera Types & Camera Models

Camera Types

- Static/Fixed Camera

Camera Types

- Wide angle and Fish-eye camera



Camera Types

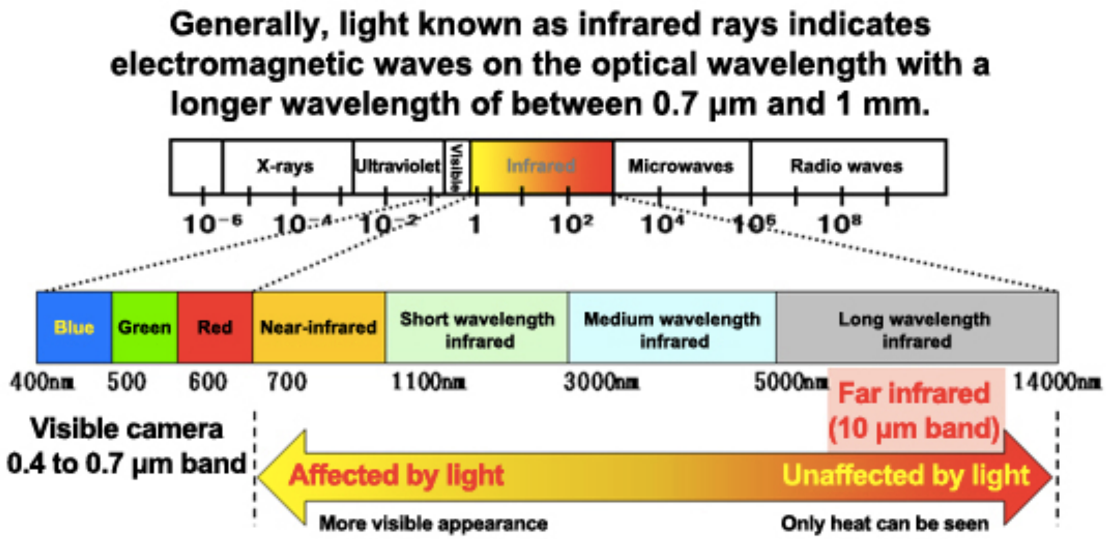
- Omni-directional camera

Camera Types

- Pan-Tilt-Zoom (PTZ) camera
- Speed-Dome Camera

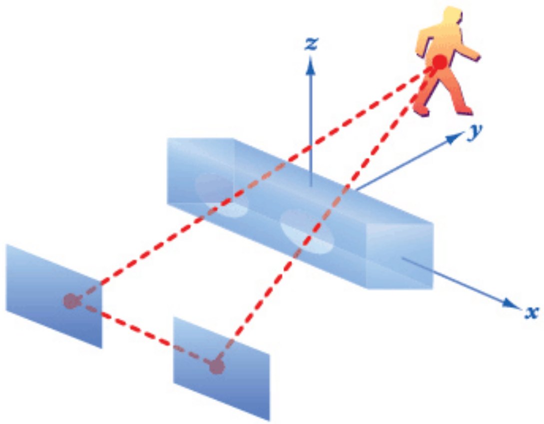
Camera Types

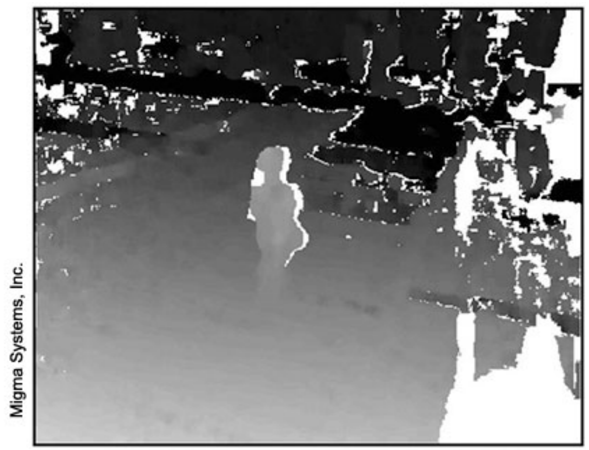
- Infrared (IR) Camera



Camera Types

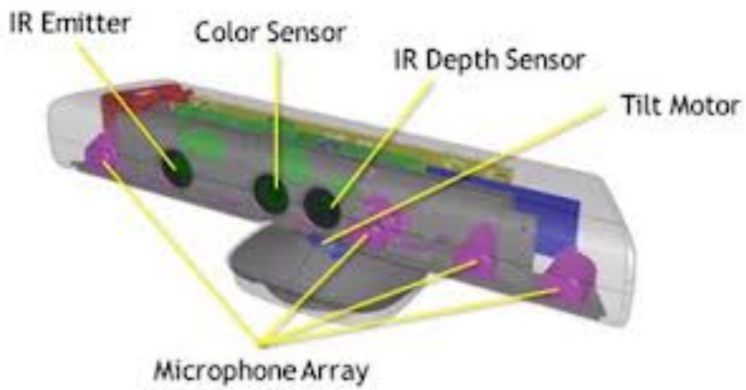
- Stereo camera





Camera Types

- Infrared-based depth camera

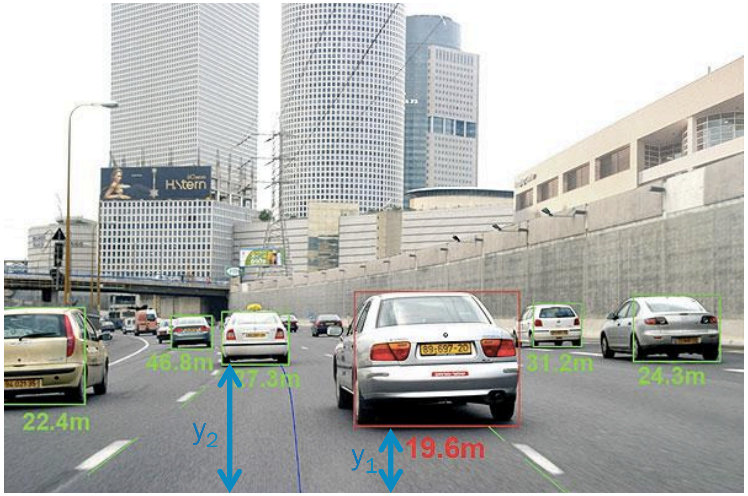


Motivation

- Getting more 3D information from images

Motivation

- Getting more 3D information from images



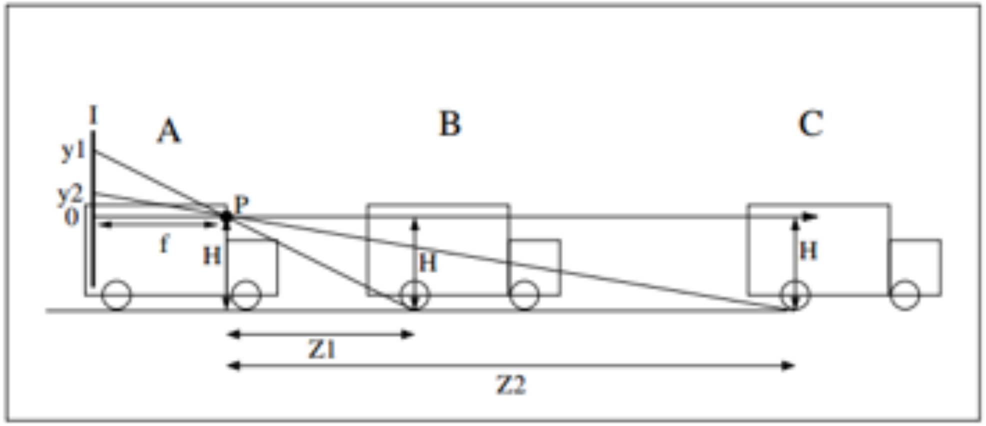
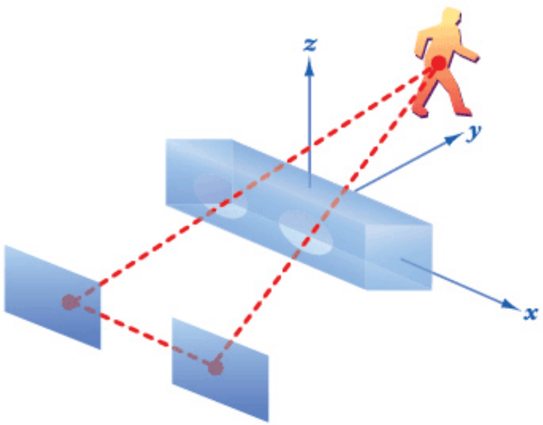
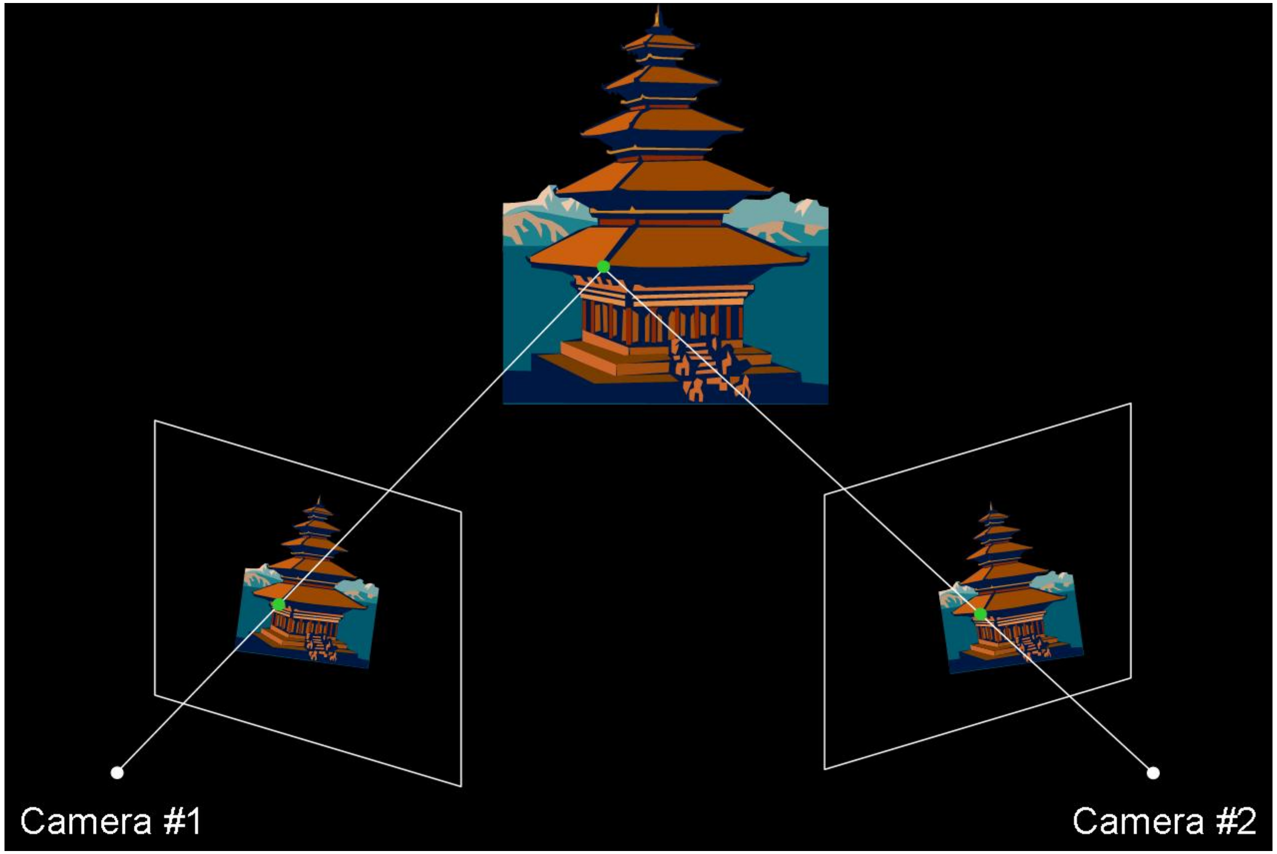


Figure 2: *Schematic diagram of the imaging geometry (see text).*

Motivation

- Getting more 3D information from images



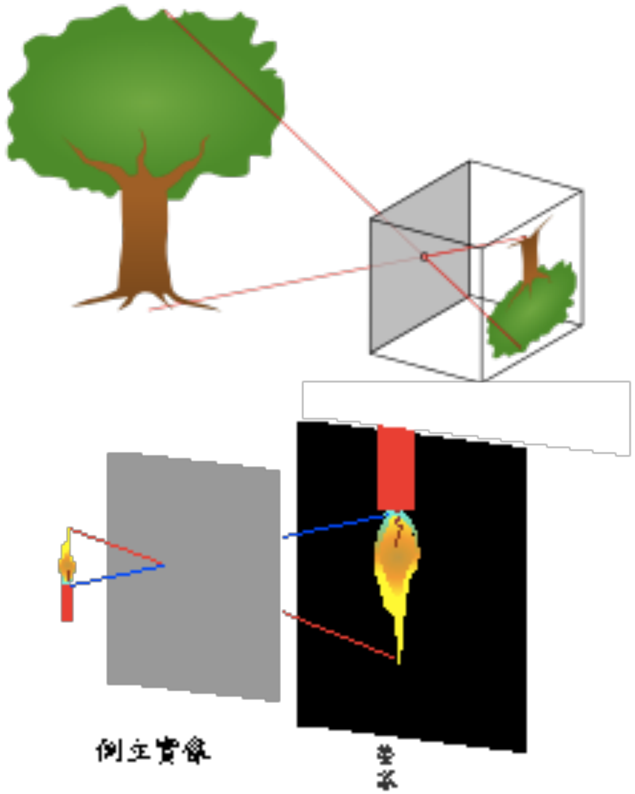


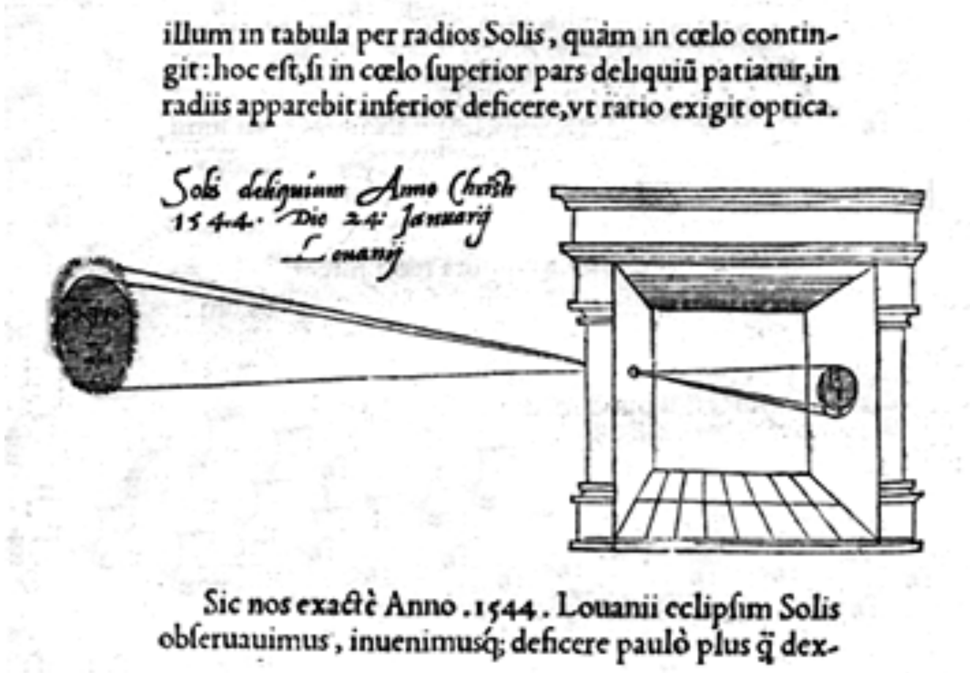
Motivation

- Integrate multiple views

Camera Projection Model

- **Pinhole camera** - also known as camera obscura, or "dark chamber”, is a simple camera without a lens and with a single small aperture, a pinhole – effectively a light-proof box with a small hole in one side.



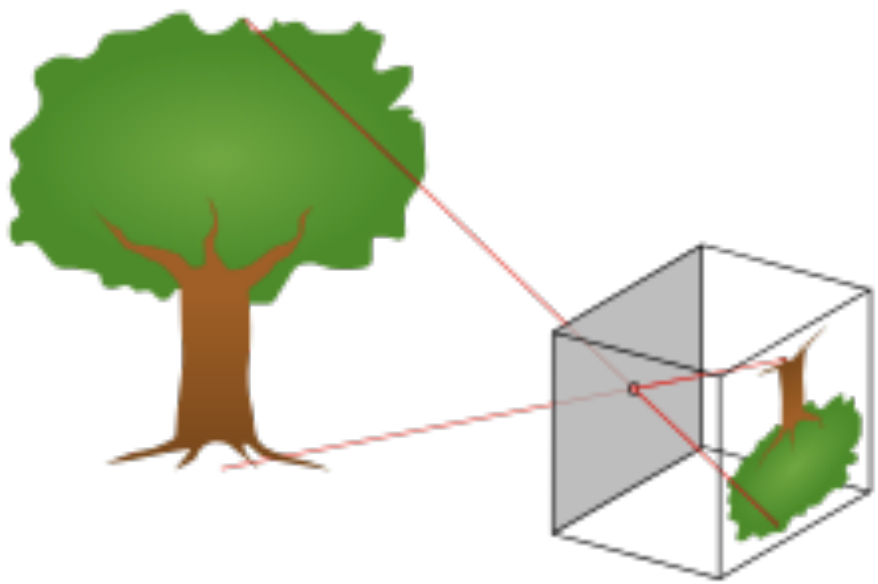

illum in tabula per radios Solis, quàm in cœlo contingit: hoc est, si in cœlo superior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

Sic nos exactè Anno .1544. Louanii eclipsim Solis obseruauimus, inuenimusq; deficere paulò plus q̃ dex-

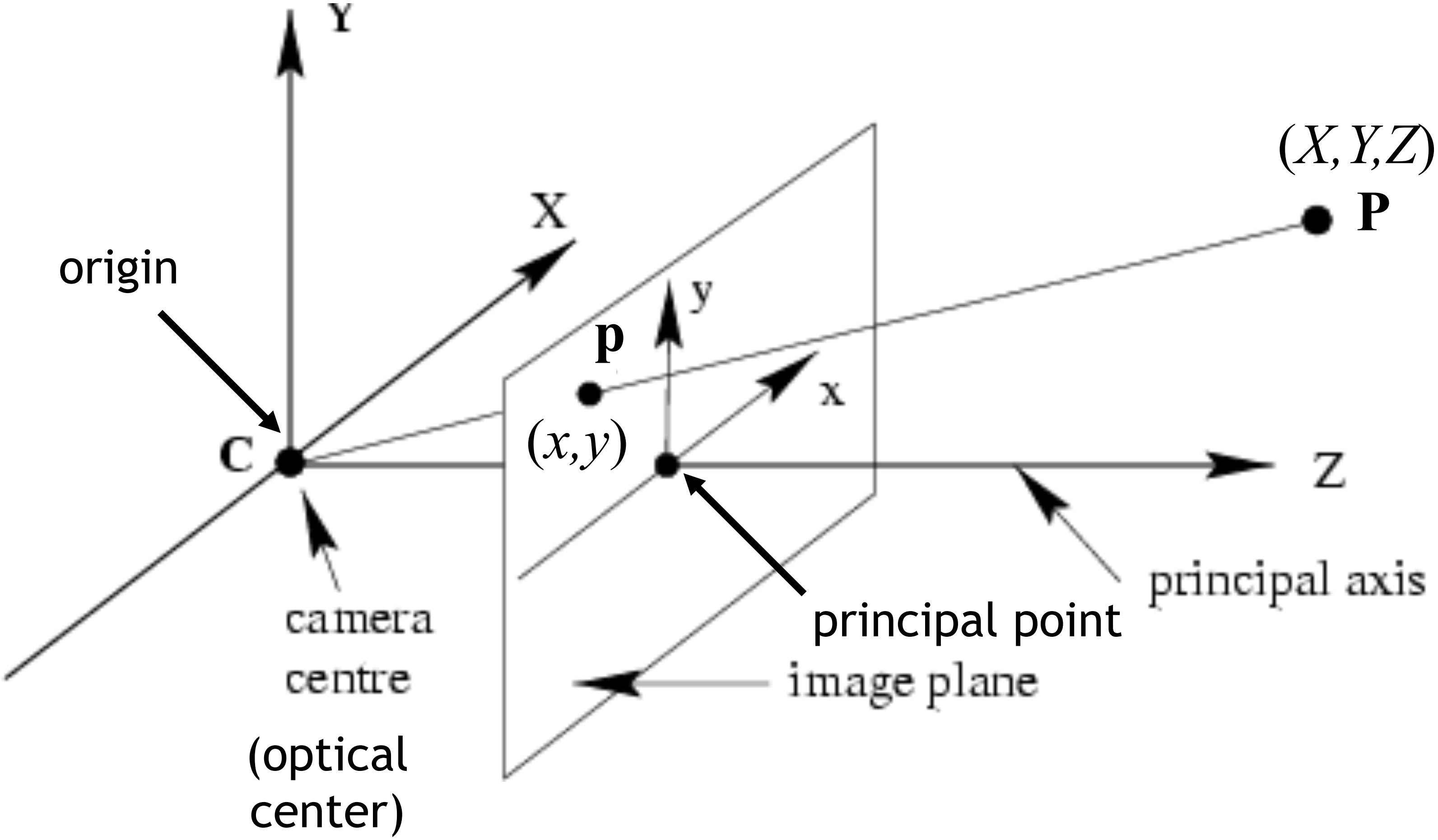
Camera Projection Model

- **Pinhole camera model** - describes the mathematical relationship between the coordinates of a 3D point and its projection onto the image plane of an ideal pinhole camera, where the camera aperture is described as a point and no lenses are used to focus light.

The model does not include geometric distortions or blurring of unfocused objects caused by lenses and finite sized apertures.

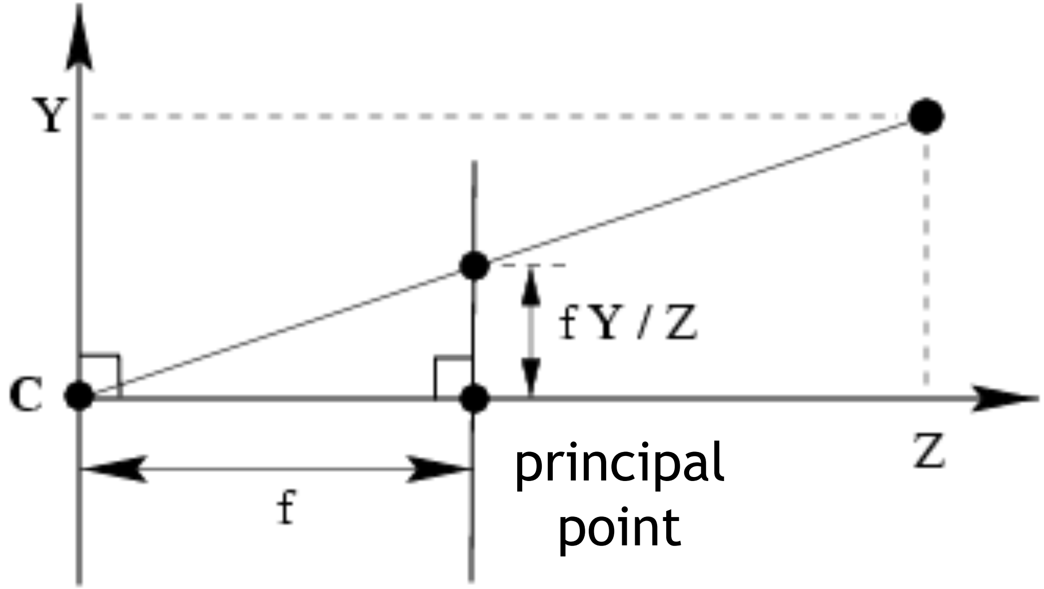


Pinhole Camera Model



Slide credit: Yung-Yu Chuang

Pinhole Camera Model



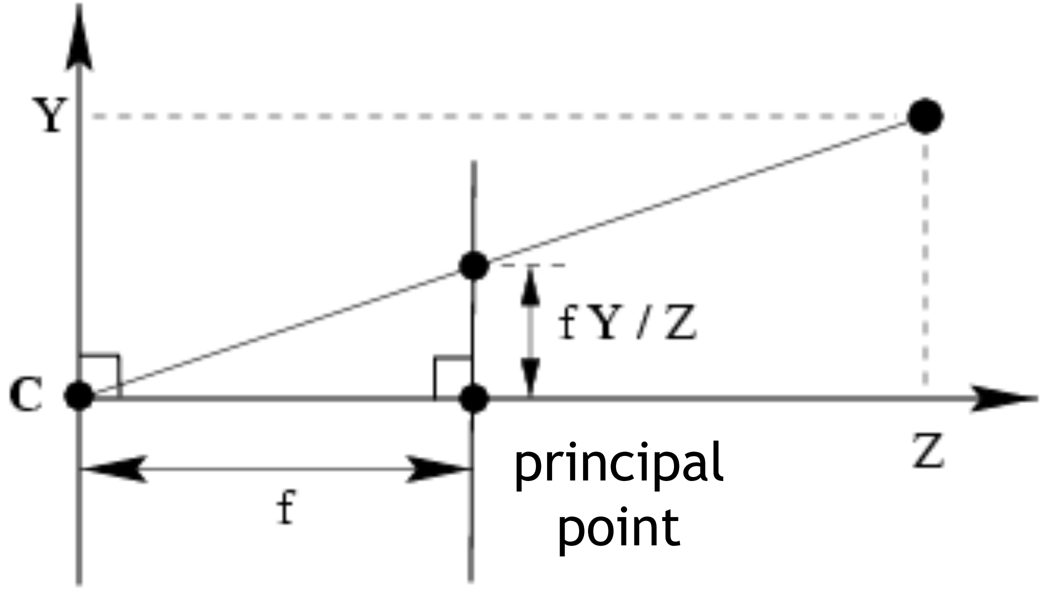
$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Slide credit: Yung-Yu Chuang

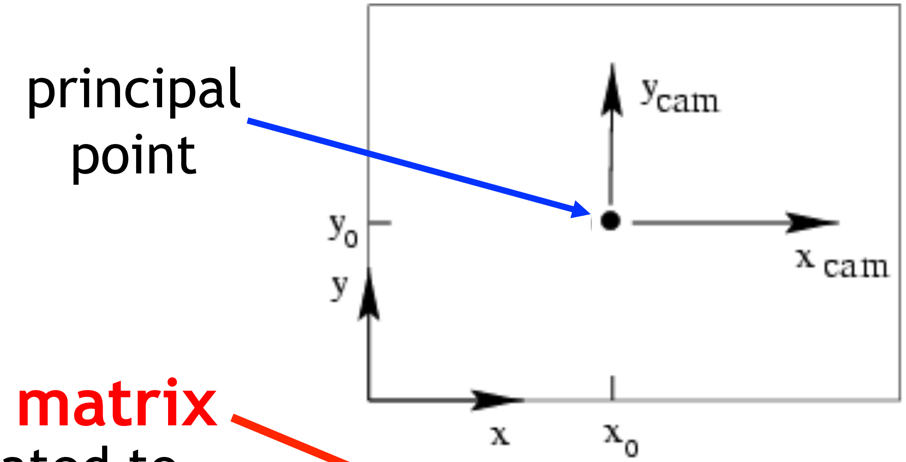
Pinhole Camera Model



$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Slide credit: Yung-Yu Chuang

Principal point offset



intrinsic matrix
only related to
camera projection

$$\mathbf{x} \sim \mathbf{K}[\mathbf{I}|0]\mathbf{X}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Slide credit: Yung-Yu Chuang

Intrinsic Matrix

Is this form of $\mathbf{K}$ good enough?

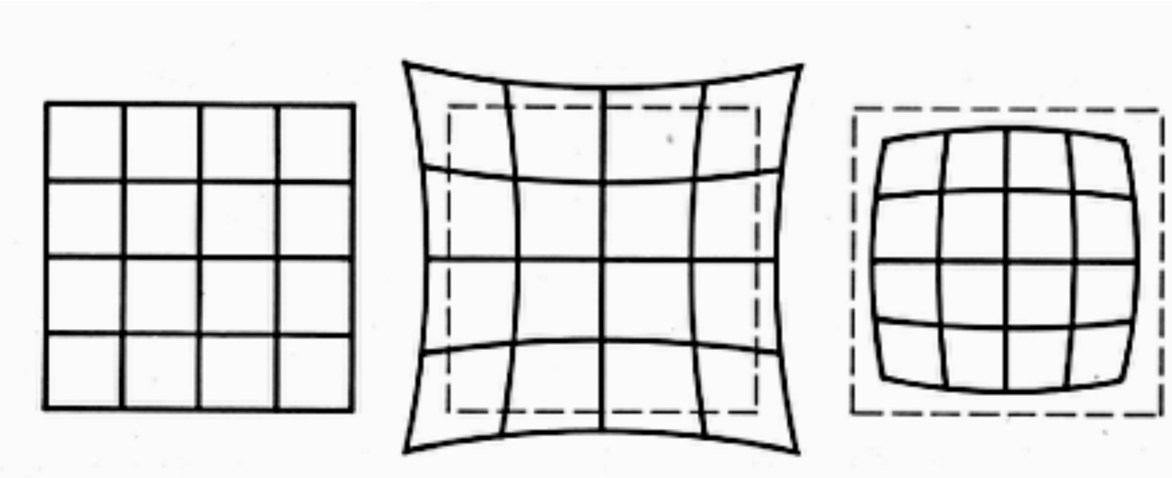
$$\mathbf{K} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

- non-square pixels (digital video)
- skew
- radial distortion

$$\mathbf{K} = \begin{bmatrix} fa & s & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Slide credit: Yung-Yu Chuang

Distortion



No distortion   Pin cushion   Barrel

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

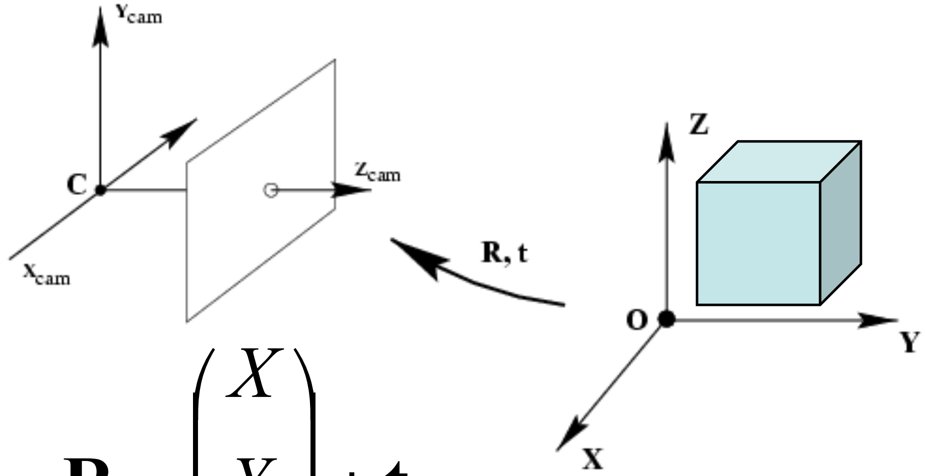
Slide credit: Yung-Yu Chuang

Distortion

$x'' = x'*(1 + k_1r^2 + k_2r^4) + 2*p_1x'*y' + p_2(r^2+2*x'^2)$
$y'' = y'*(1 + k_1r^2 + k_2r^4) + p_1(r^2+2*y'^2) + 2*p_2*x'*y'$
where $r^2 = x'^2+y'^2$

dewarp

Camera rotation and translation



$$\begin{pmatrix}X'\\Y'\\Z'\end{pmatrix} = \mathbf{R}_{3\times3}\begin{pmatrix}X\\Y\\Z\end{pmatrix}+\mathbf{t}$$

$$\begin{pmatrix}x\\y\\1\end{pmatrix}\sim\begin{bmatrix}f&0&x_0\\0&f&y_0\\0&0&1\end{bmatrix}\left[\mathbf{R}\middle|\mathbf{t}\right]\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}$$

$$\mathbf{x}\sim\mathbf{K}\underbrace{\left[\mathbf{R}\middle|\mathbf{t}\right]}_{\text{extrinsic matrix}}\mathbf{X}$$

Slide credit: Yung-Yu Chuang

Two kinds of parameters

- internal or intrinsic parameters such as focal length, optical center, aspect ratio:
  - what kind of camera?
- external or extrinsic (pose) parameters including rotation and translation:
  - where is the camera?

Intrinsic parameters

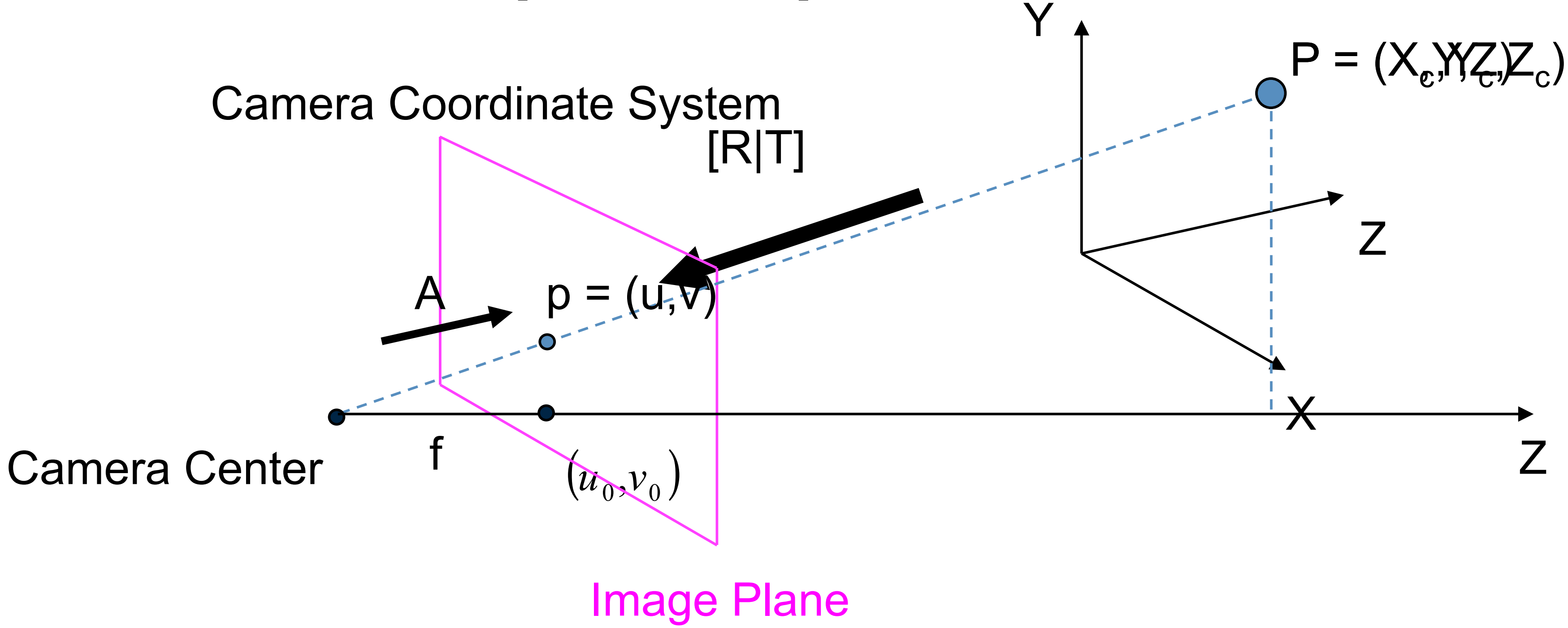
$$\mathbf{x} \sim \mathbf{K}\underbrace{\left[\mathbf{R} \middle| \mathbf{t}\right]}_{\text{Extrinsic parameters}}\mathbf{X}$$

Slide credit: Yung-Yu Chuang

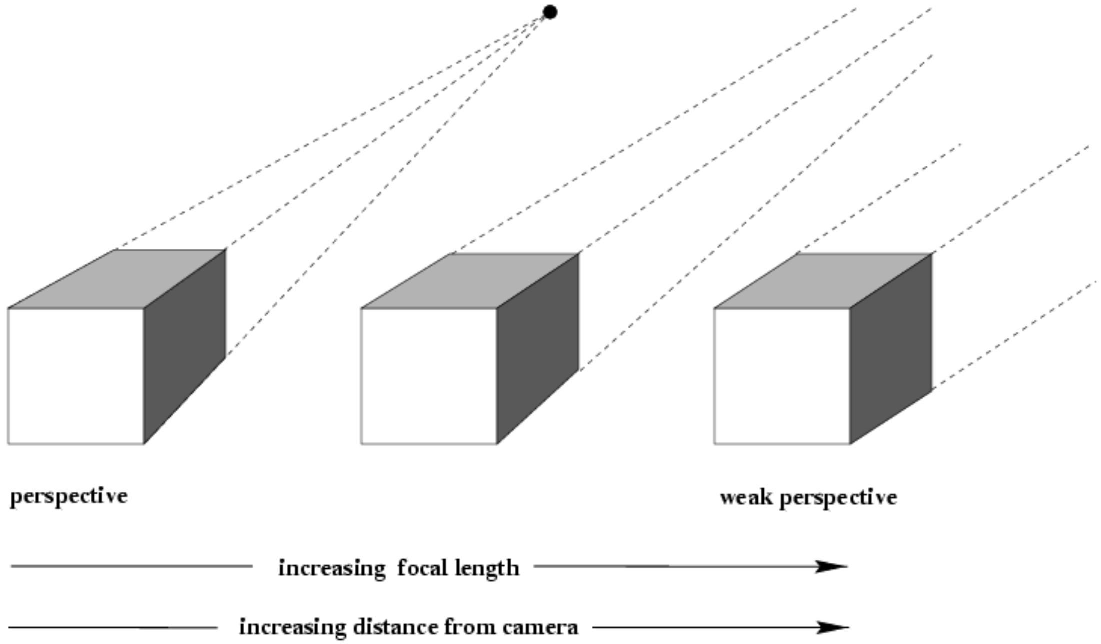
Camera Parameters

$$
\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f/du & \gamma & u_0 \\ 0 & f/dv & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}
$$

Global Coordinate System



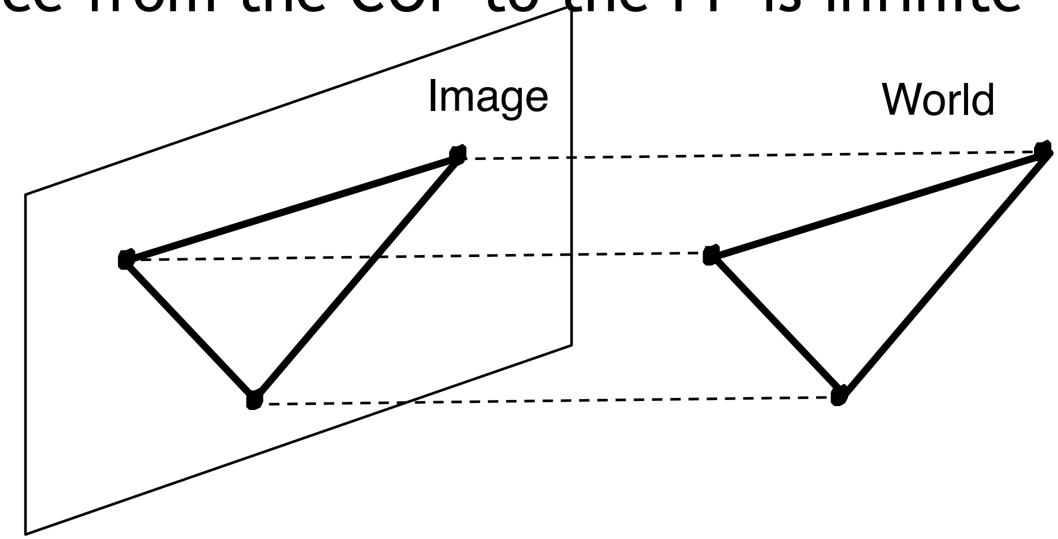
Other projection models



Slide credit: Yung-Yu Chuang

Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

  - Also called “parallel projection”: (x, y, z) $\rightarrow$ (x, y)

Slide credit: Yung-Yu Chuang

Other types of projections

- Scaled orthographic
  - Also called “weak perspective”

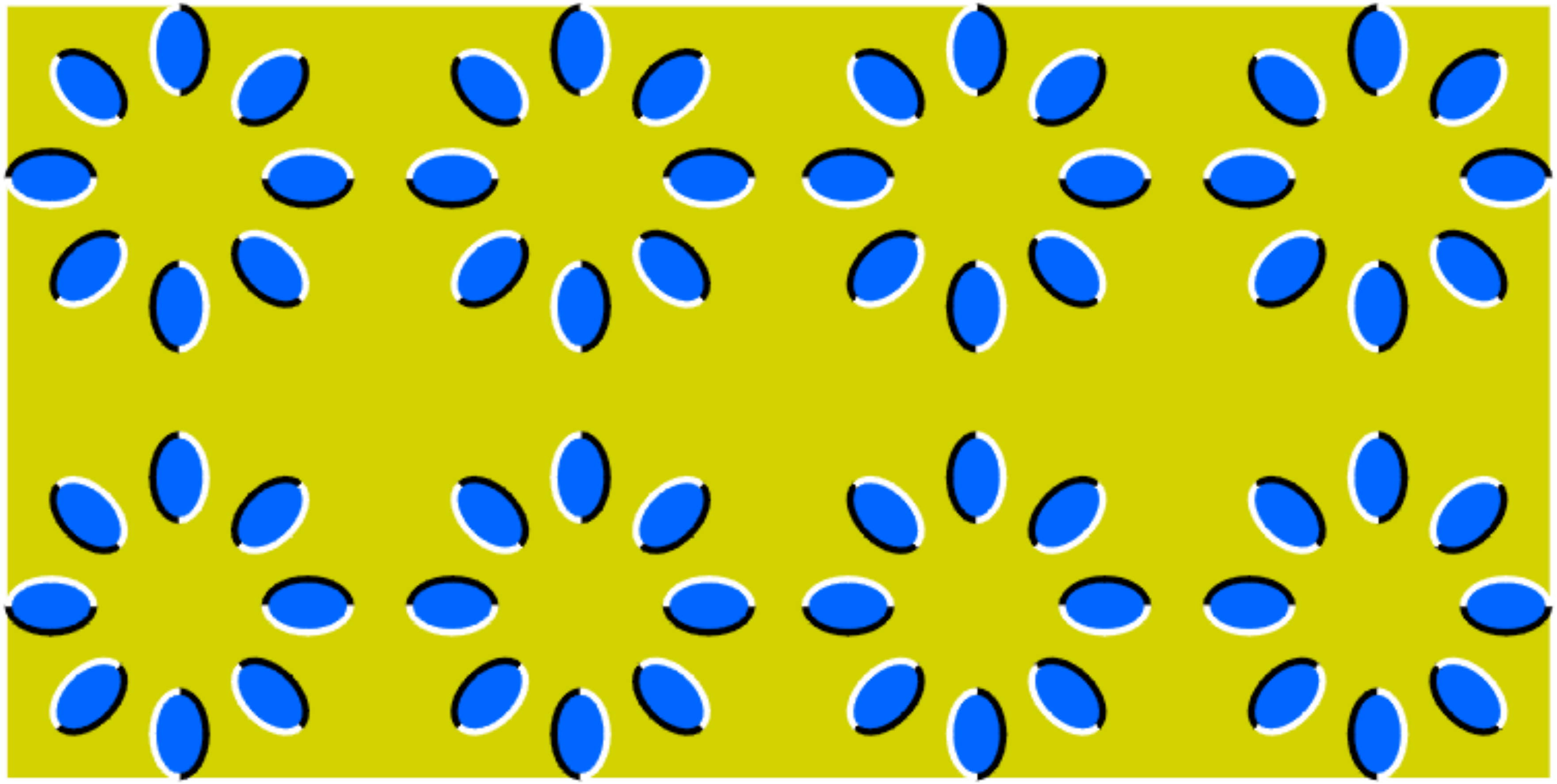
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

- Affine projection
  - Also called “paraperspective”

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

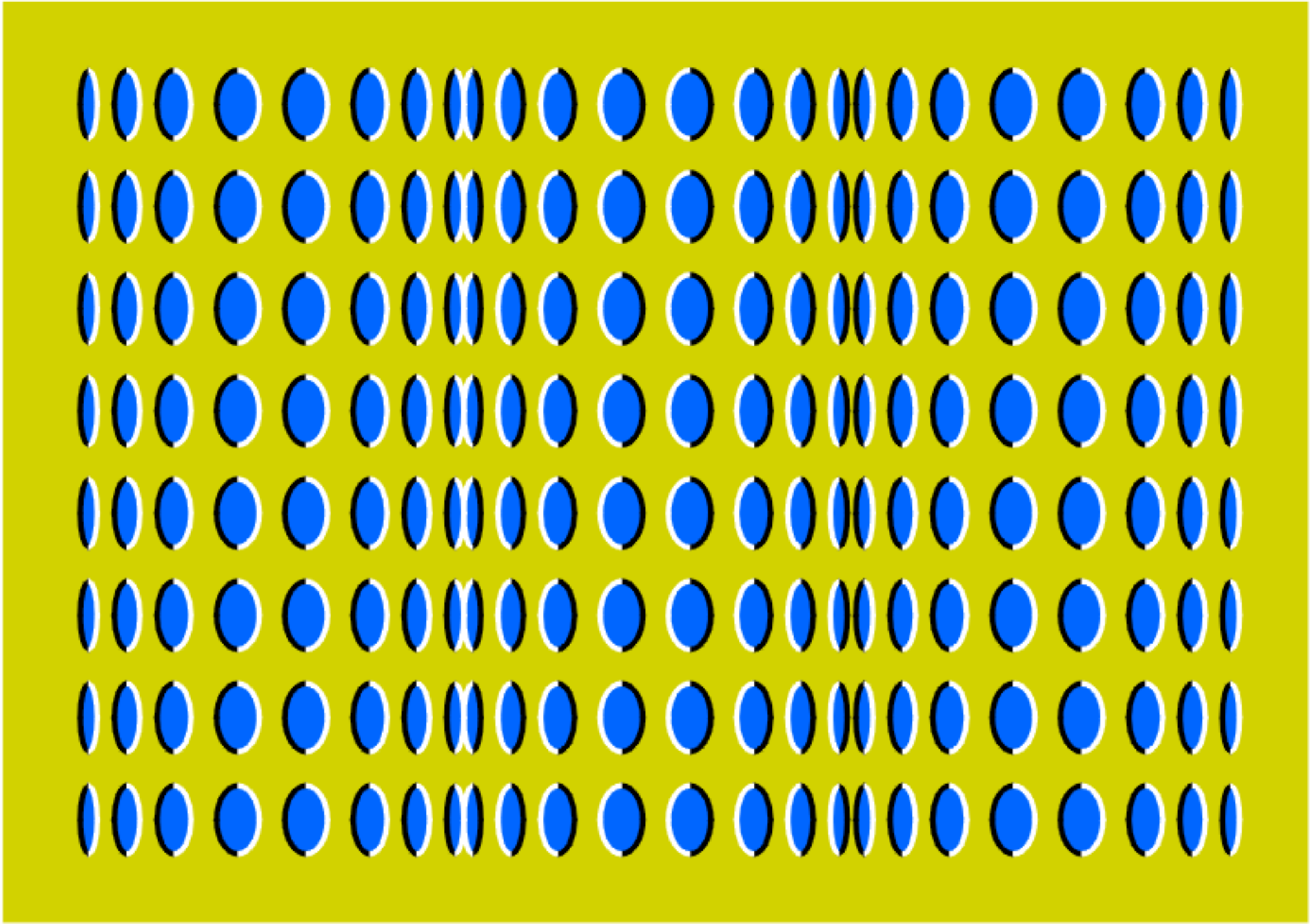
Slide credit: Yung-Yu Chuang

Illusion



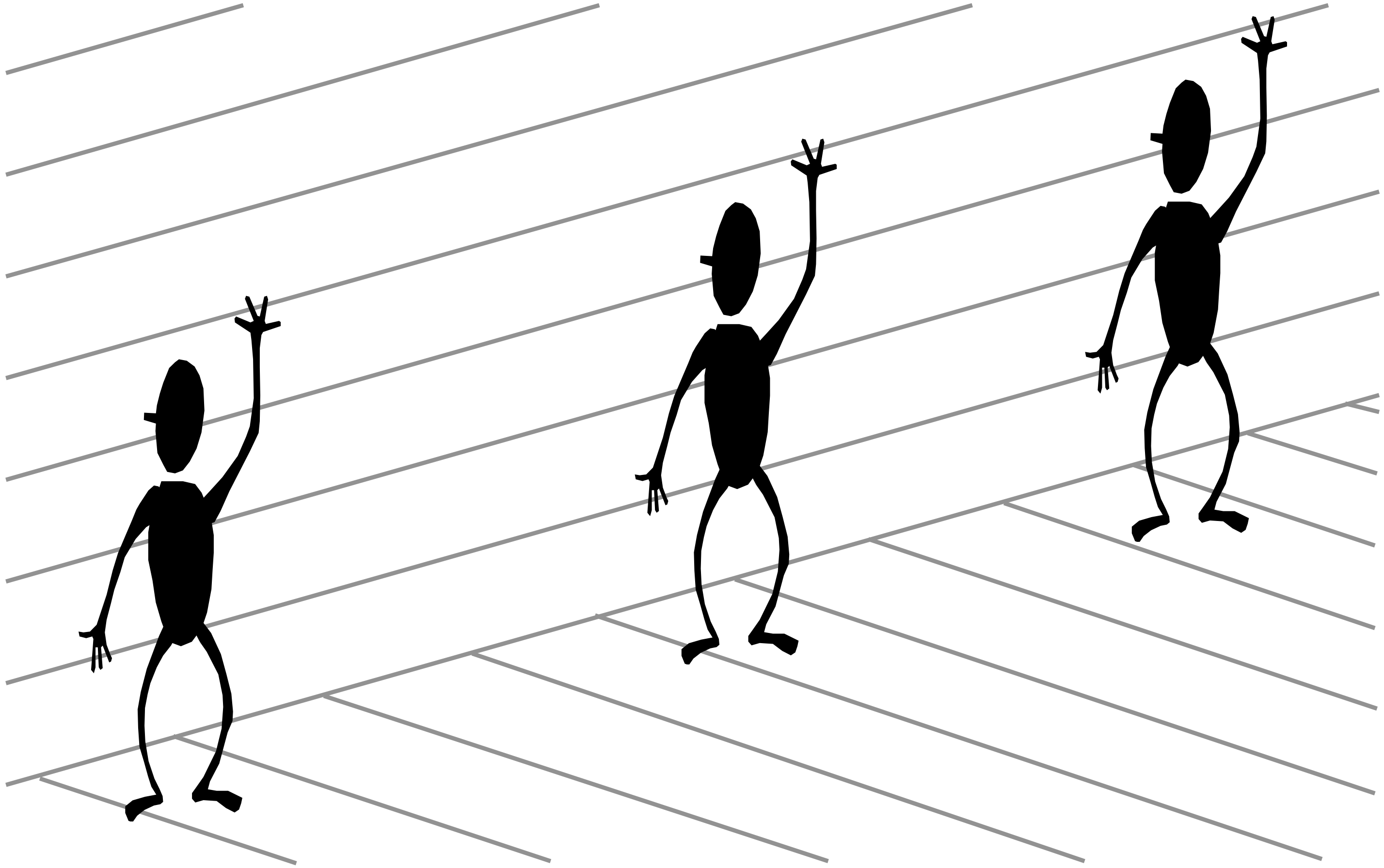
Slide credit: Yung-Yu Chuang

Illusion



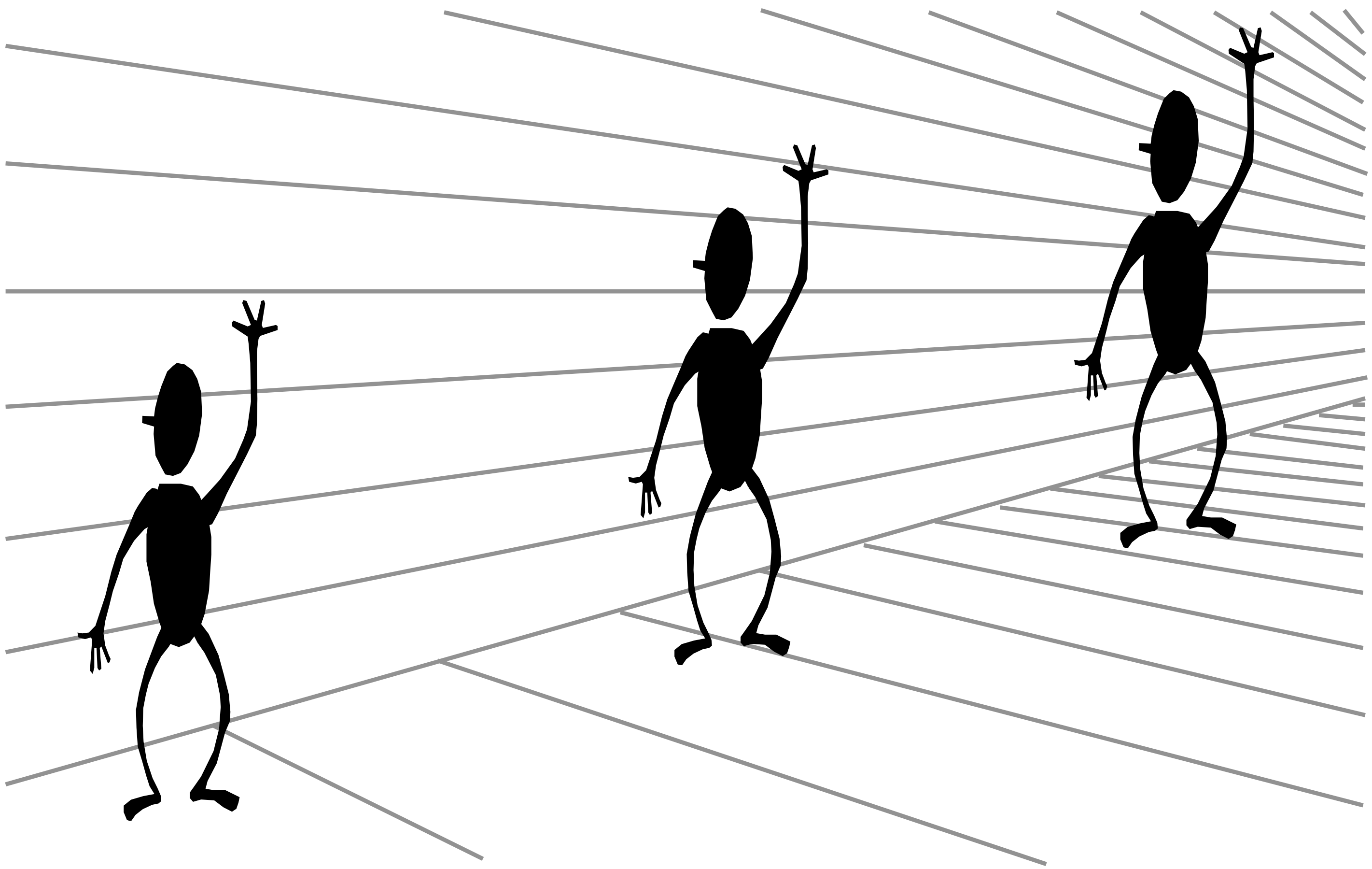
Slide credit: Yung-Yu Chuang

Fun with perspective



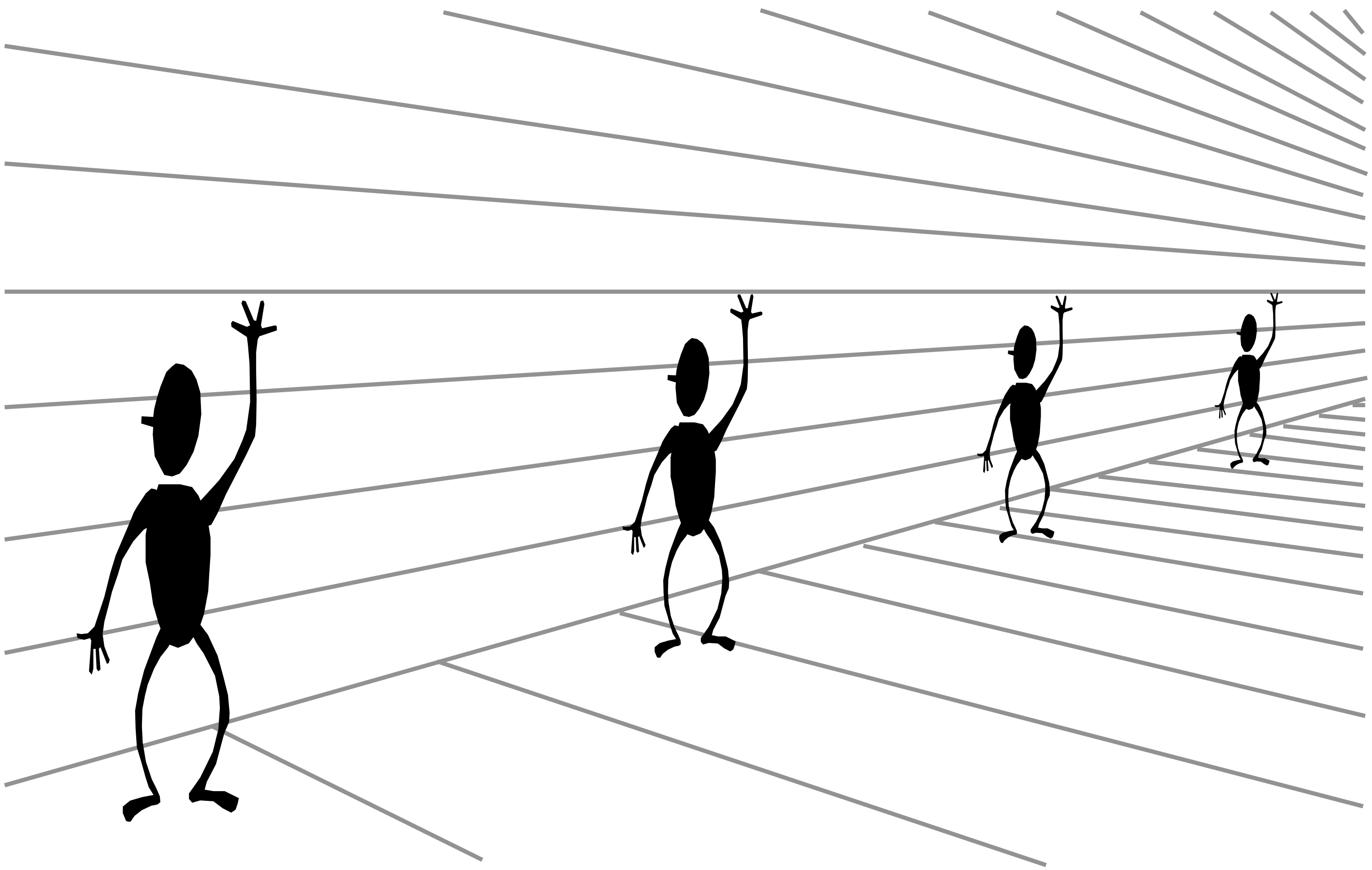
Slide credit: Yung-Yu Chuang

Fun with perspective



Slide credit: Yung-Yu Chuang

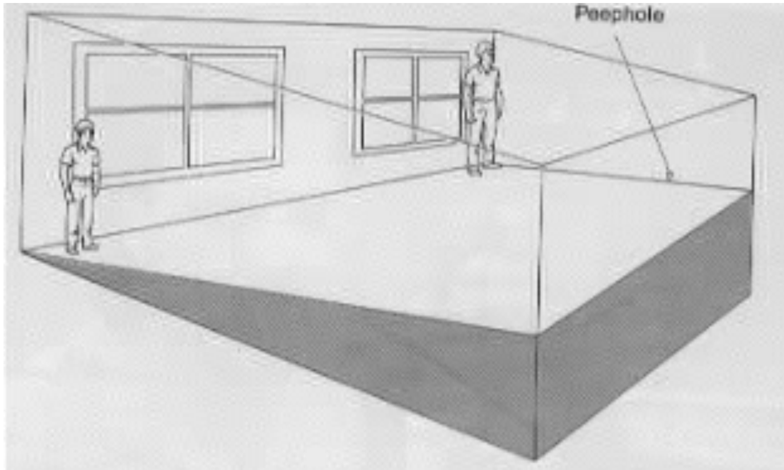
Fun with perspective

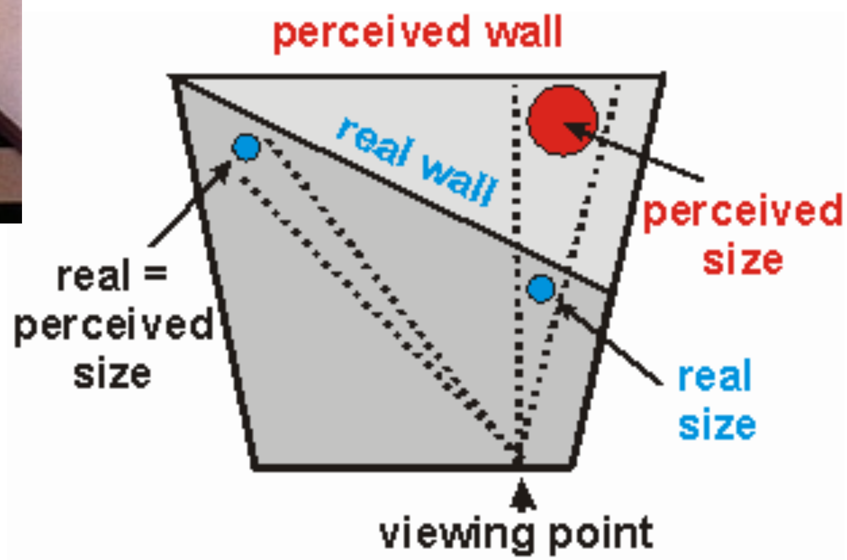


Slide credit: Yung-Yu Chuang

Fun with perspective

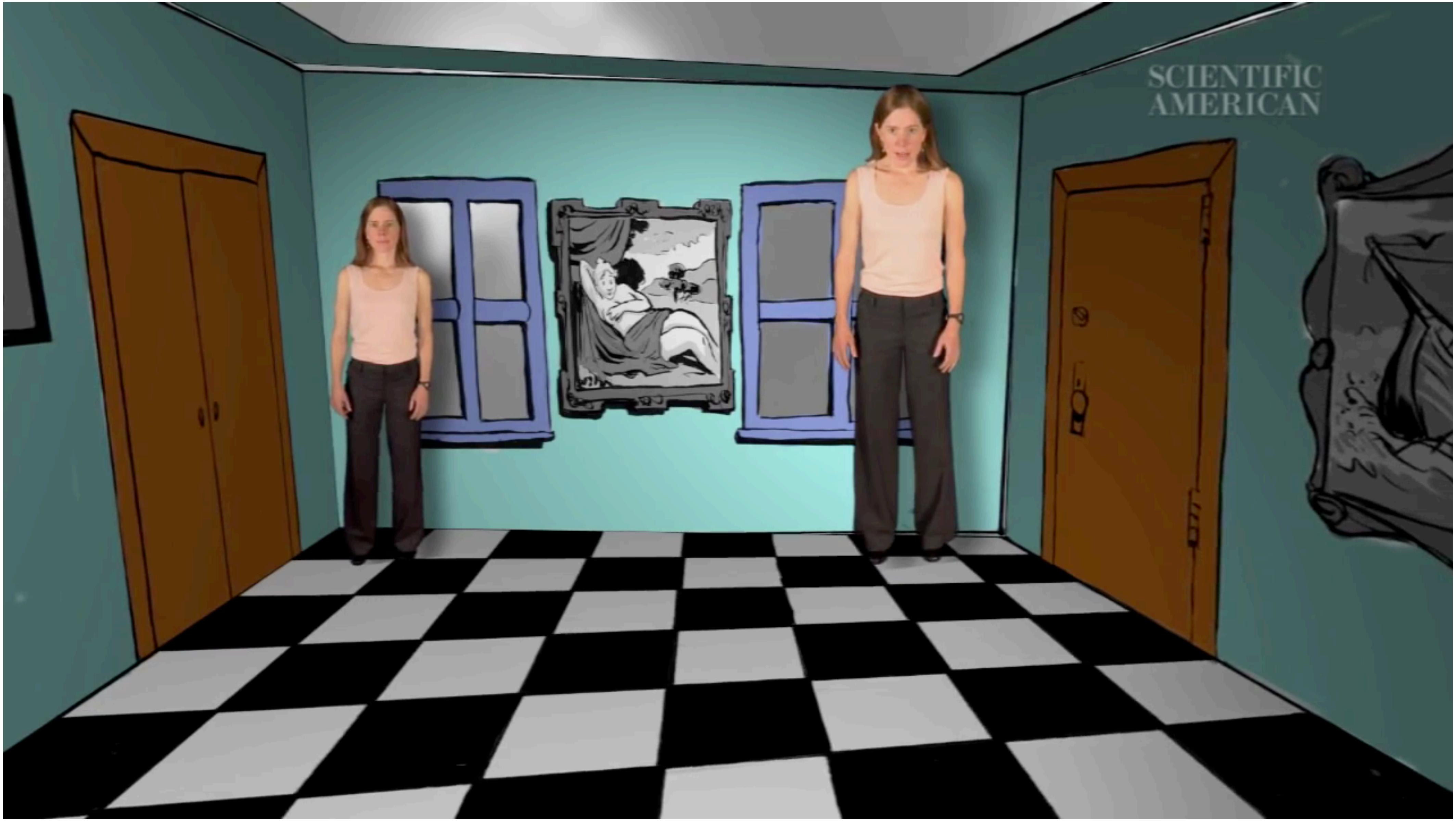
Ames room





Slide credit: Yung-Yu Chuang

Ames room



Forced perspective in LOTR

Two kinds of parameters

- internal or intrinsic parameters such as focal length, optical center, aspect ratio:
  - what kind of camera?
- external or extrinsic (pose) parameters including rotation and translation:
  - where is the camera?

$$\mathbf{x} \sim \overbrace{\mathbf{K}}^{\text{Intrinsic parameters}} \underbrace{[\mathbf{R}|\mathbf{t}]}_{\text{Extrinsic parameters}} \mathbf{X}$$

Slide credit: Yung-Yu Chuang

Camera Calibration

Camera Calibration

- Estimate both intrinsic and extrinsic parameters.
- Two main categories:
  - **Photometric calibration**: uses reference objects (3D, 2D, 1D, 0D) with known geometry
  - **Self calibration**: only assumes static scene, e.g. structure from motion

Intrinsic parameters

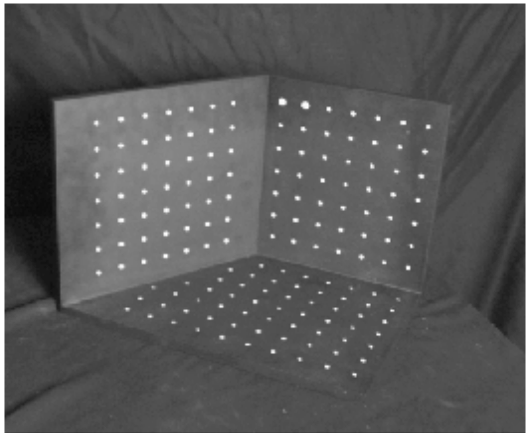
$$\mathbf{x} \sim \mathbf{K}\underbrace{[\mathbf{R}|\mathbf{t}]}_{\text{Extrinsic parameters}}\mathbf{X}$$

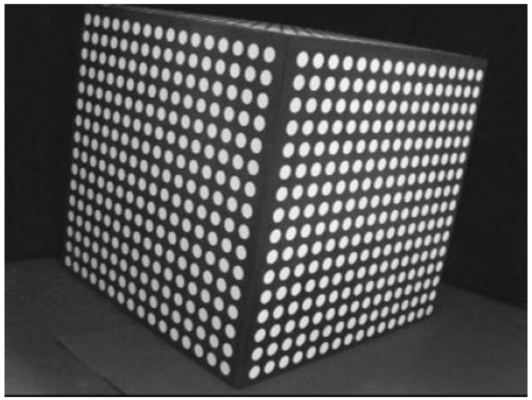
Slide credit: Yung-Yu Chuang

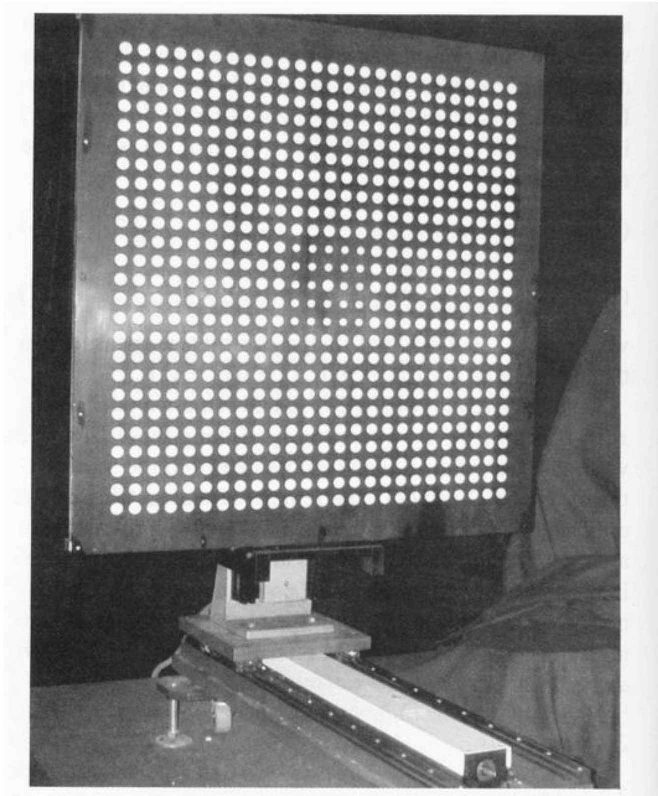
Camera Calibration

- Known 2D coordinates in the image and their corrresponding 3D coordinates in the world, then we can solve the parameters by
  - linear regression (least squares)
  - nonlinear optimization

$$\mathbf{x} \sim \overset{\text{Intrinsic parameters}}{\mathbf{K}} \underbrace{[\mathbf{R}|\mathbf{t}]}_{\text{Extrinsic parameters}} \mathbf{X}$$





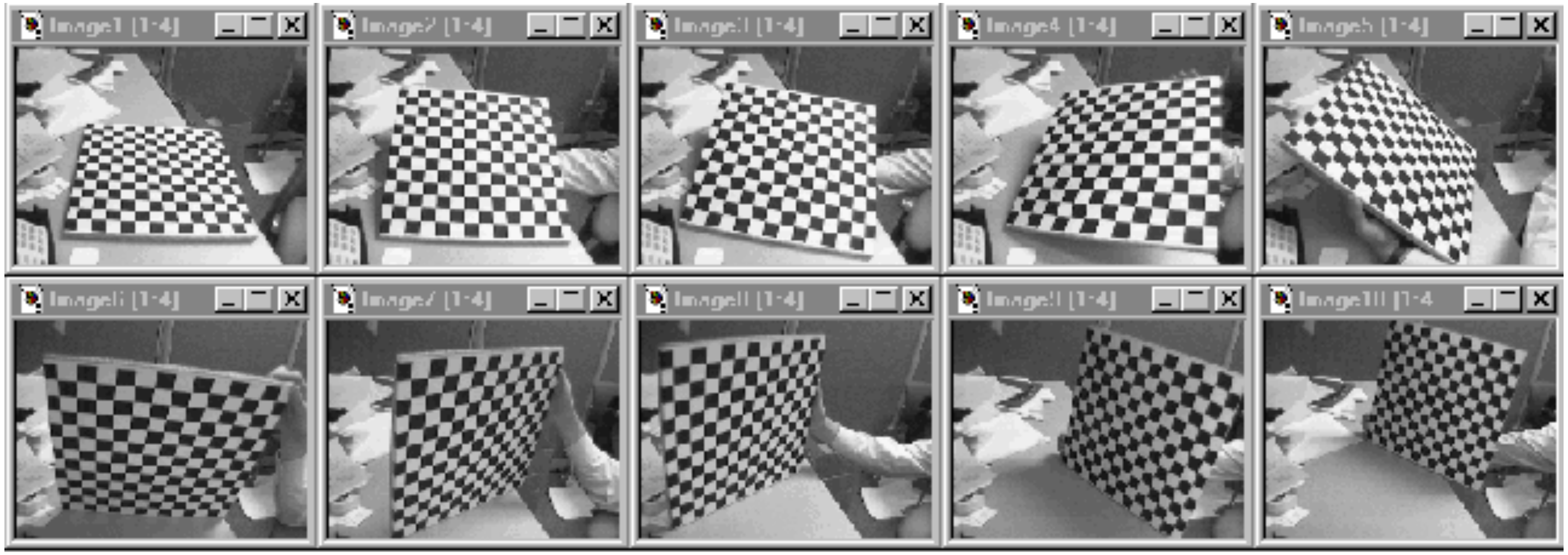


Camera Calibration

Z. Zhang, "Flexible Camera Calibration by Viewing a Plane from Unknown Orientations," *International Conference on Computer Vision (ICCV)*, 1999.
(cited number: 2561 from Google)

Z. Zhang, "A flexible new technique for camera calibration," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2000.
(cited number: 9781 from Google)

Multi-plane calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don't have to know positions/orientations
- Good code available online!
  - Intel's OpenCV library: http://www.intel.com/research/mrl/research/opencv/
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Zhengyou Zhang's web site: http://research.microsoft.com/~zhang/Calib/

Slide credit: Yung-Yu Chuang

Notation

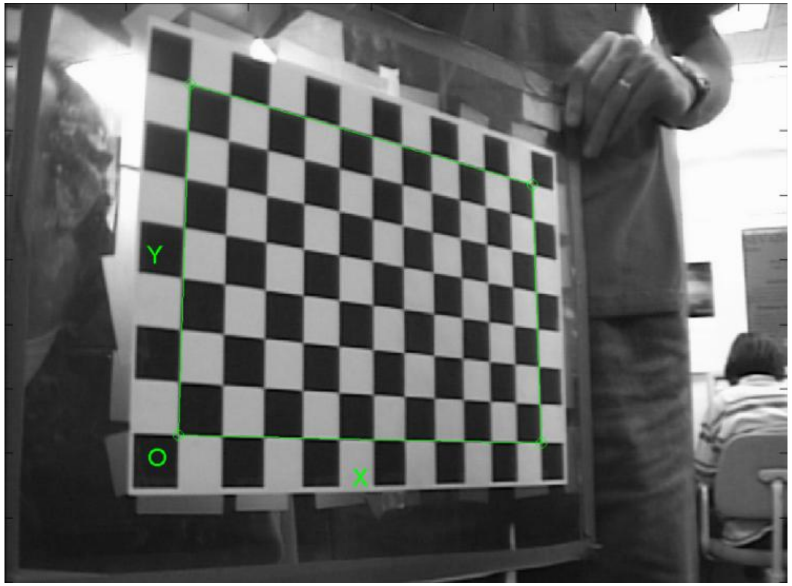
$$\mathbf{x} \sim \mathbf{K}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}$$

- 2D point : $m = [u, v]^T \longrightarrow \widetilde{m} = [u, v, 1]^T$
- 3D point : $M = [X, Y, Z]^T \longrightarrow \widetilde{M} = [X, Y, Z, 1]^T$
- The usual pinhole :

$$s\widetilde{m} = A[R \mid t]\widetilde{M} \quad \text{, with} \quad A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Using the abbreviation A^{-T} for $(A^{-1})^T$ or $(A^T)^{-1}$

Estimating Homography



?

Homography

- Projective transformation

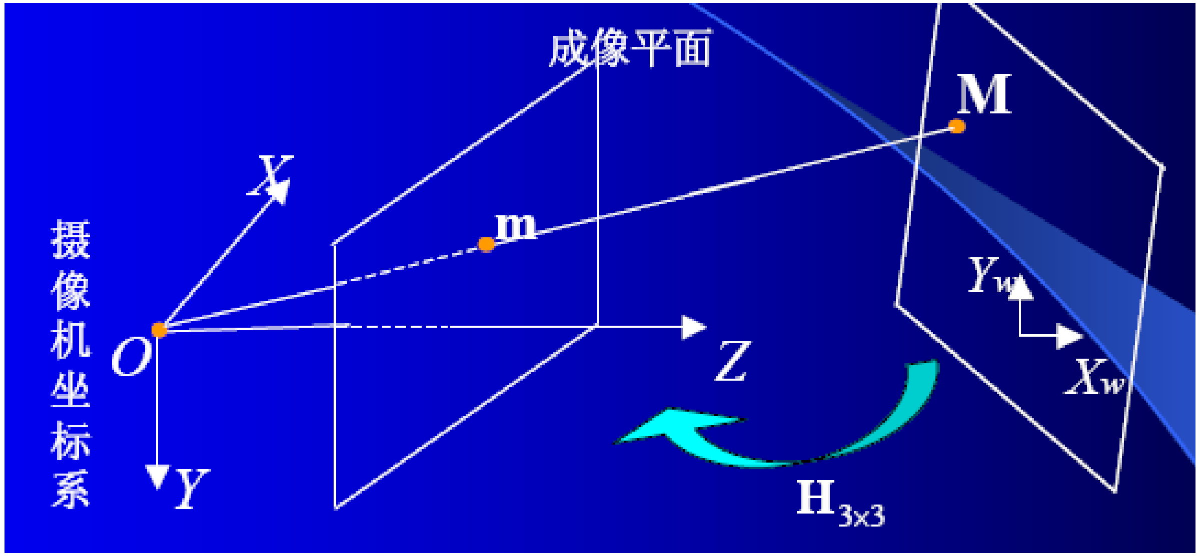
- Defined in 2D space as a mapping between a point on a ground plane as seen from one camera, to the same point on the ground plane as seen from a second camera

$$s\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}}_{\textbf{Homography}}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

8 unknowns
 - at least 4 points are needed

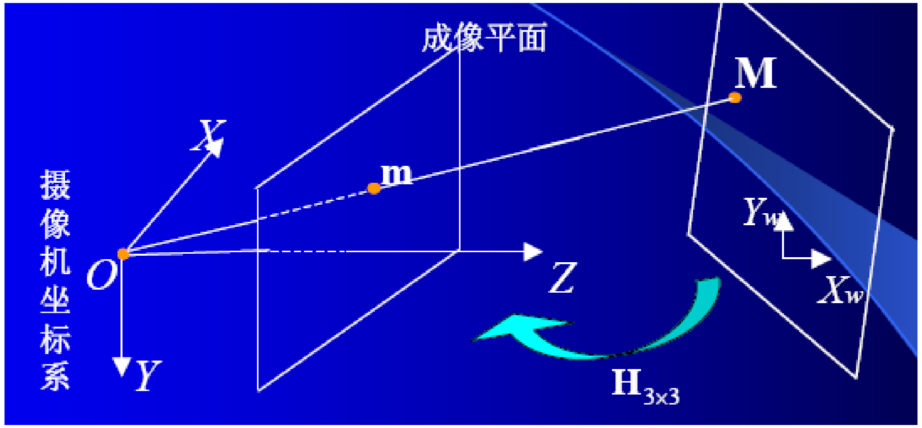
Homography - Case 1

- Case 1: a mapping between image coordinates and ground plane coordinates



Proof – Case 1

$$s\tilde{m} = A[R \mid t]\tilde{M}$$



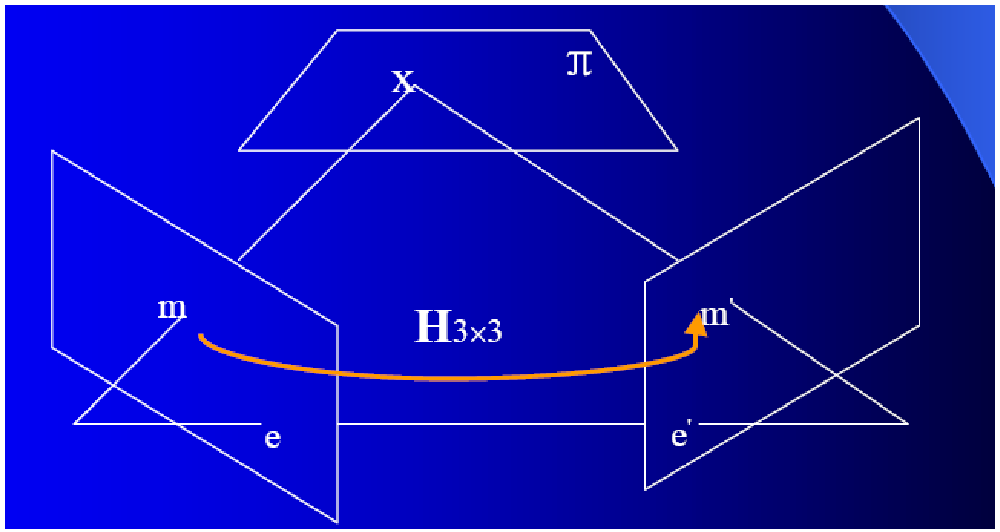
- We assume the model plane is on Z = 0, then

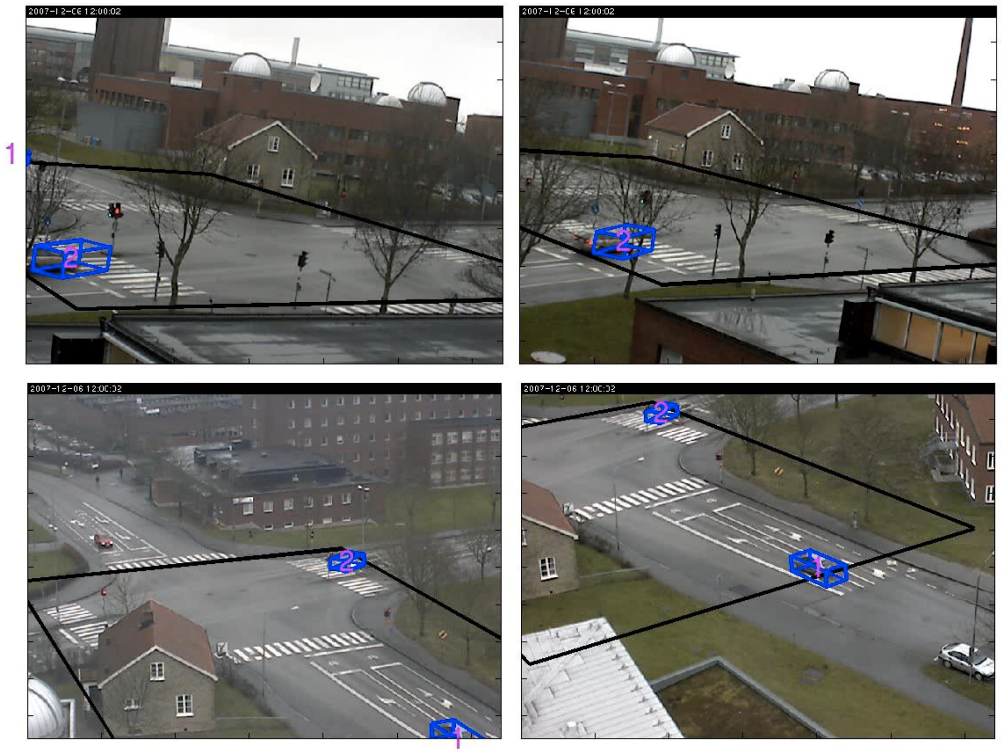
$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ 0 \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ 1 \end{bmatrix}$$

$$s\tilde{\mathbf{m}} = \mathbf{H}\tilde{\mathbf{M}}, \quad \text{with } \mathbf{H} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

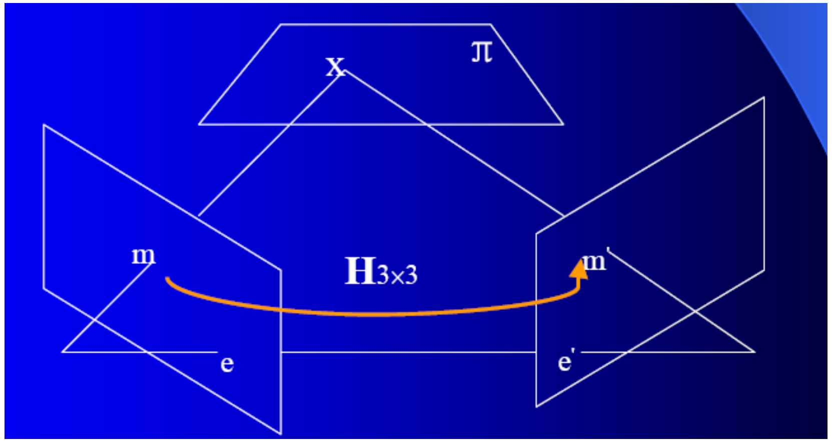
Homography - Case 2

- Case 1: a mapping between a point on a ground plane as seen from one camera, to the same point on the ground plane as seen from a second camera





Proof – Case 2



- m_1 is the image coordinate of Camera 1
- m_2 is the image coordinate of Camera 2
- M is the coordinate on the ground plane
- From Case 1:

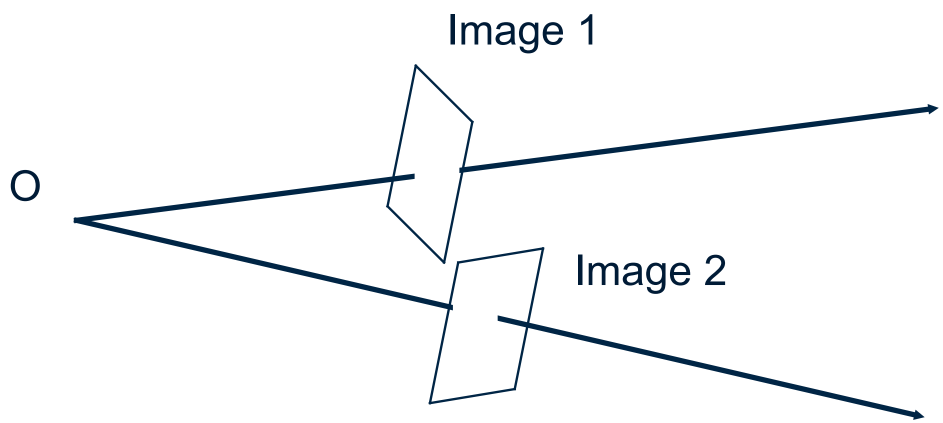
$$s_1\tilde{\mathbf{m}}_1 = \mathbf{H}_1\tilde{\mathbf{M}}$$

$$s_2\tilde{\mathbf{m}}_2 = \mathbf{H}_2\tilde{\mathbf{M}} \longrightarrow s\tilde{\mathbf{m}}_2 = \mathbf{H}_2\mathbf{H}_1^{-1}\tilde{\mathbf{m}}_1$$

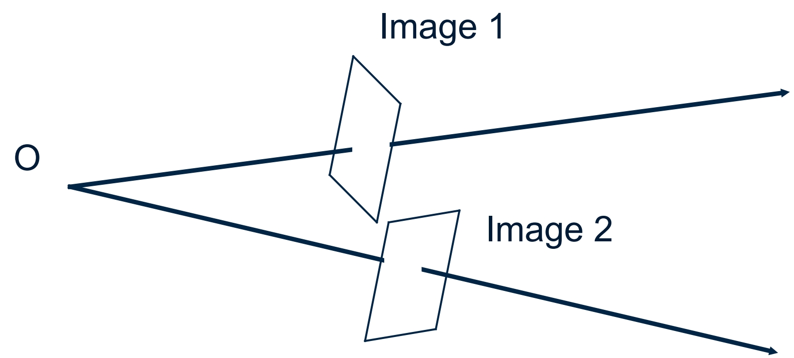
$$\longrightarrow s\tilde{\mathbf{m}}_2 = \mathbf{H}\tilde{\mathbf{m}}_1$$

Homography - Case 3

- Case 1: a mapping between image coordinates of Camera 1 and image coordinates of Camera 2, where Camera 1 and Camera 2 is located in the same position



Proof – Case 3

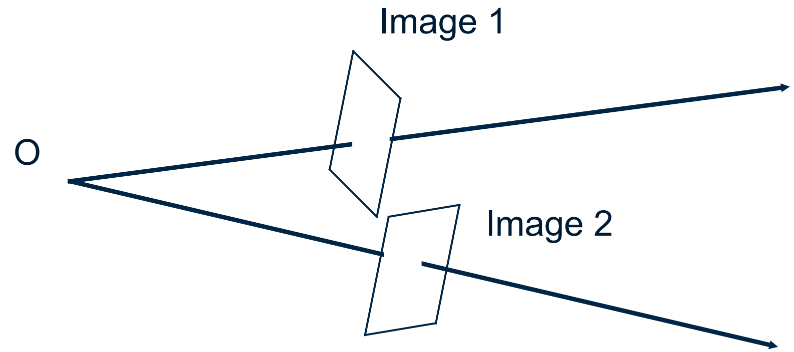


$$s\widetilde{m} = A[R \mid t]\widetilde{M}$$

- Let the position of camera is (0, 0, 0) in the global coordinate system, then

$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & 0 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{A}R\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Proof – Case 3



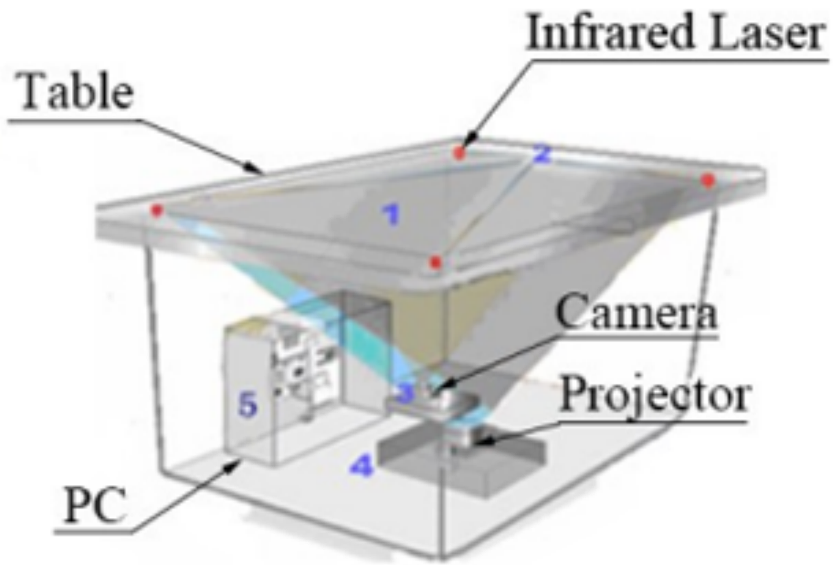
$$s_1 \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{A}R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \cdots\cdots\cdots \quad \textbf{(1)}$$

$$s_2 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A}R' \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \cdots\cdots\cdots \quad \textbf{(2)}$$

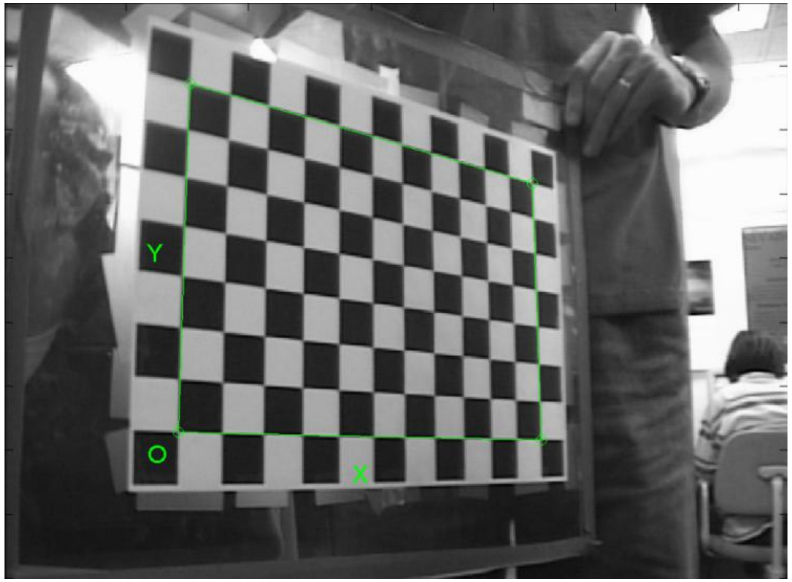
From (1), (2) :

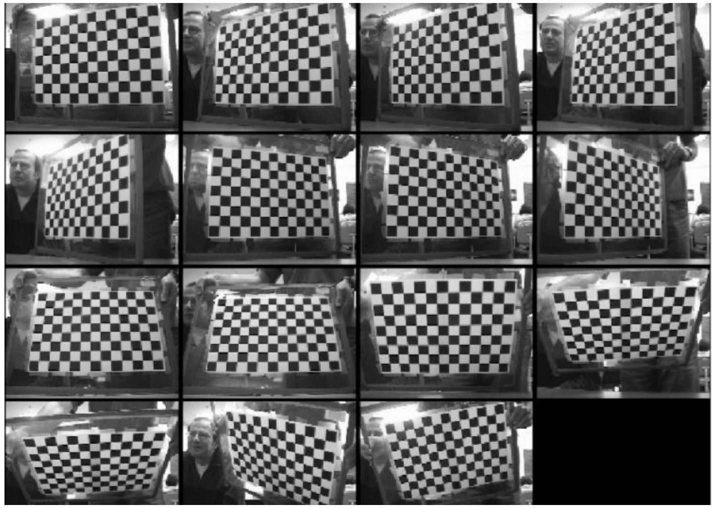
$$s \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A}R'R\mathbf{A}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Application of Homography - Projector-Camera System

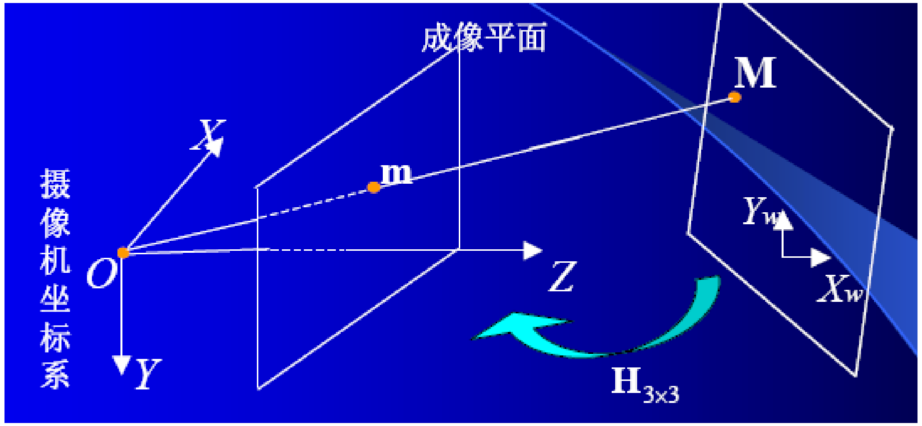


Estimating Homography





- Case 1: a mapping between image coordinates and ground plane coordinates



$$s\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Estimating Homography

- Without loss of generality, we assume the model plane is on Z = 0

$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ 0 \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ 1 \end{bmatrix}$$

$$s\widetilde{\mathbf{m}} = \mathbf{H}\widetilde{\mathbf{M}}, \quad \text{with } \mathbf{H} = \mathbf{A}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

Constraints on Intrinsic Parameters

$$\mathbf{H} = \mathbf{A}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}$$

- Denote $\mathbf{H} = \begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3\end{bmatrix}$

  then $\begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3\end{bmatrix} = \lambda\mathbf{A}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix} \Rightarrow \begin{matrix}\mathbf{r}_1 = \mathbf{A}^{-1}\mathbf{h}_1 \\ \mathbf{r}_2 = \mathbf{A}^{-1}\mathbf{h}_2\end{matrix}$

- Since r_1 and r_2 are orthonormal

$$\begin{cases}\|\mathbf{r}_1\| = \|\mathbf{r}_2\| \\ \mathbf{r}_1 \cdot \mathbf{r}_2 = 0\end{cases} \Longrightarrow \begin{cases}\mathbf{h}_1^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_2 \\ \mathbf{h}_1^T\underline{\mathbf{A}^{-T}\mathbf{A}^{-1}}\mathbf{h}_2 = 0\end{cases}$$

Closed-Form Solution

- Let

$$A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{B} = \mathbf{A}^{-T}\mathbf{A}^{-1}$$

$$= \begin{bmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} \\ \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} & \frac{(v_0\gamma - u_0\beta)^2}{\alpha^2\beta^2} + \frac{v_0^2}{\beta^2} + 1 \end{bmatrix}$$

Closed-Form Solution

- B is symmetric, defined by a 6D vector

$$\mathbf{b} = \begin{bmatrix} B_{11} & B_{12} & B_{22} & B_{13} & B_{23} & B_{33} \end{bmatrix}^T$$

- Let *i*th column vector of H be $\mathbf{h}_i = \begin{bmatrix} h_{i1} & h_{i2} & h_{i3} \end{bmatrix}^T$

$$\mathbf{h}_i^T \mathbf{B} h_j = \mathbf{v}_{ij}^T \mathbf{b}$$

with $$\mathbf{v}_{ij} = \begin{bmatrix} h_{i1}h_{j1} & h_{i1}h_{j2} + h_{i2}h_{j1} & h_{i2}h_{j2} \\ h_{i3}h_{j1} + h_{i1}h_{j3} & h_{i3}h_{j2} + h_{i2}h_{j3} & h_{i3}h_{j3} \end{bmatrix}$$

$$\boxed{\begin{cases} \mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 \\ \mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 = 0 \end{cases}}$$

Closed-Form Solution

$$\boxed{\begin{cases} \mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 \\ \mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 = 0 \end{cases}}$$

- Therefore, two constrains can be written as

$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = 0$$

- If n images of the model plane are observed

$$\mathbf{V}\mathbf{b} = 0$$

where V is a 2n × 6 matrix

Closed-Form Solution

- If n $\geqq$ 3, we will have in general a unique solution b defined up to a scale factor

- The solution is well-known as the eigenvector of V^TV associated with the smallest eigenvalue

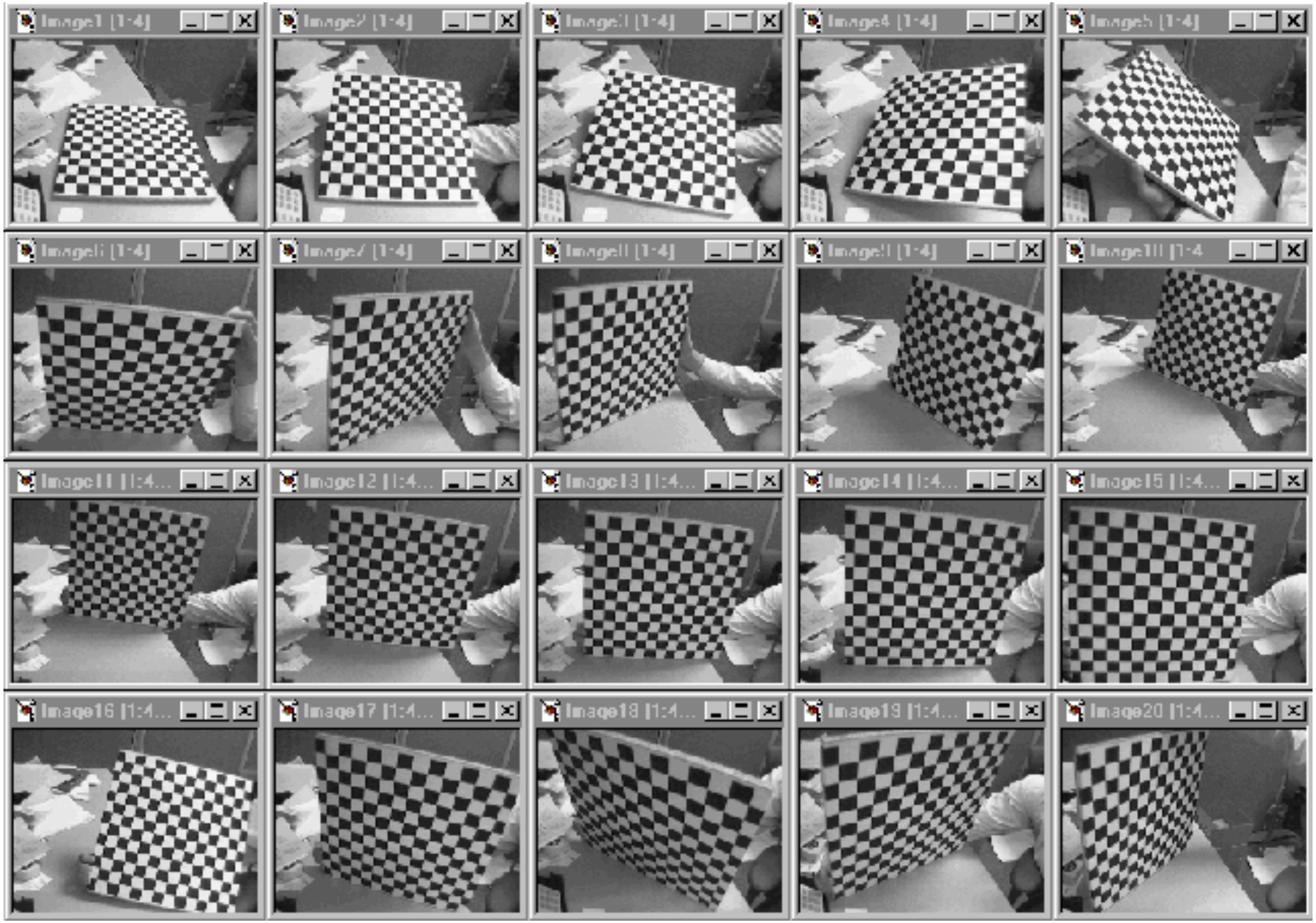
Closed-Form Solution

- Once b is estimated, then

$$
\begin{aligned}
v_0 &= \left(B_{12}B_{13} - B_{11}B_{23}\right)/\left(B_{11}B_{22} - B_{12}{}^2\right) \\
\lambda &= B_{33} - \left[B_{13}^2 + v_0\left(B_{12}B_{13} - B_{11}B_{23}\right)\right]/ B_{11} \\
\alpha &= \sqrt{\lambda / B_{11}} \\
\beta &= \sqrt{\lambda B_{11} /\left(B_{11}B_{22} - B_{12}^2\right)} \\
\gamma &= -B_{12}\alpha^2\beta / \lambda \\
u_0 &= \gamma v_0 / \beta - B_{13}\alpha^2 / \gamma
\end{aligned}
$$

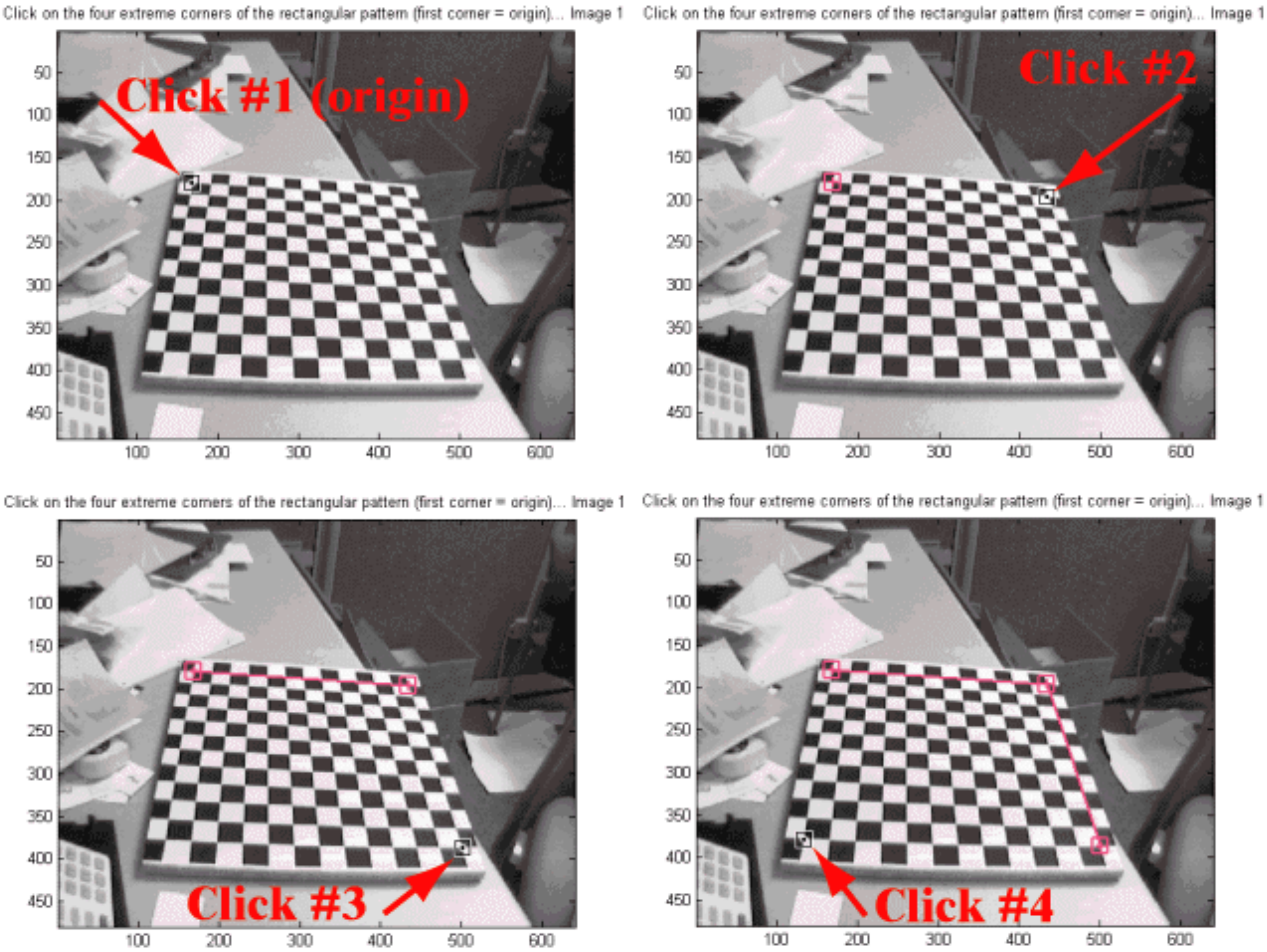
$$
\boxed{A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}
$$

Step 1: data acquisition



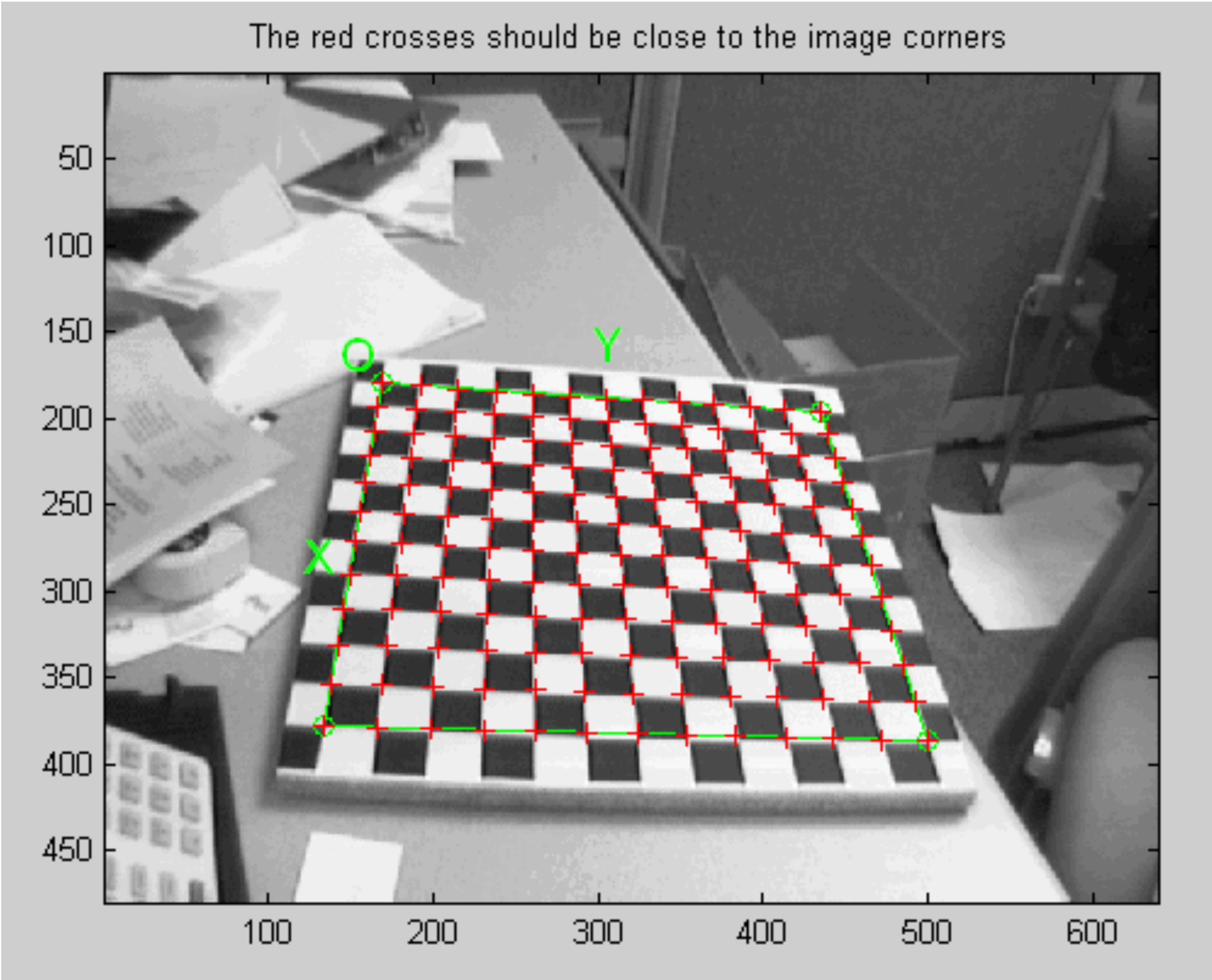
Slide credit: Yung-Yu Chuang

Step 2: specify corner order

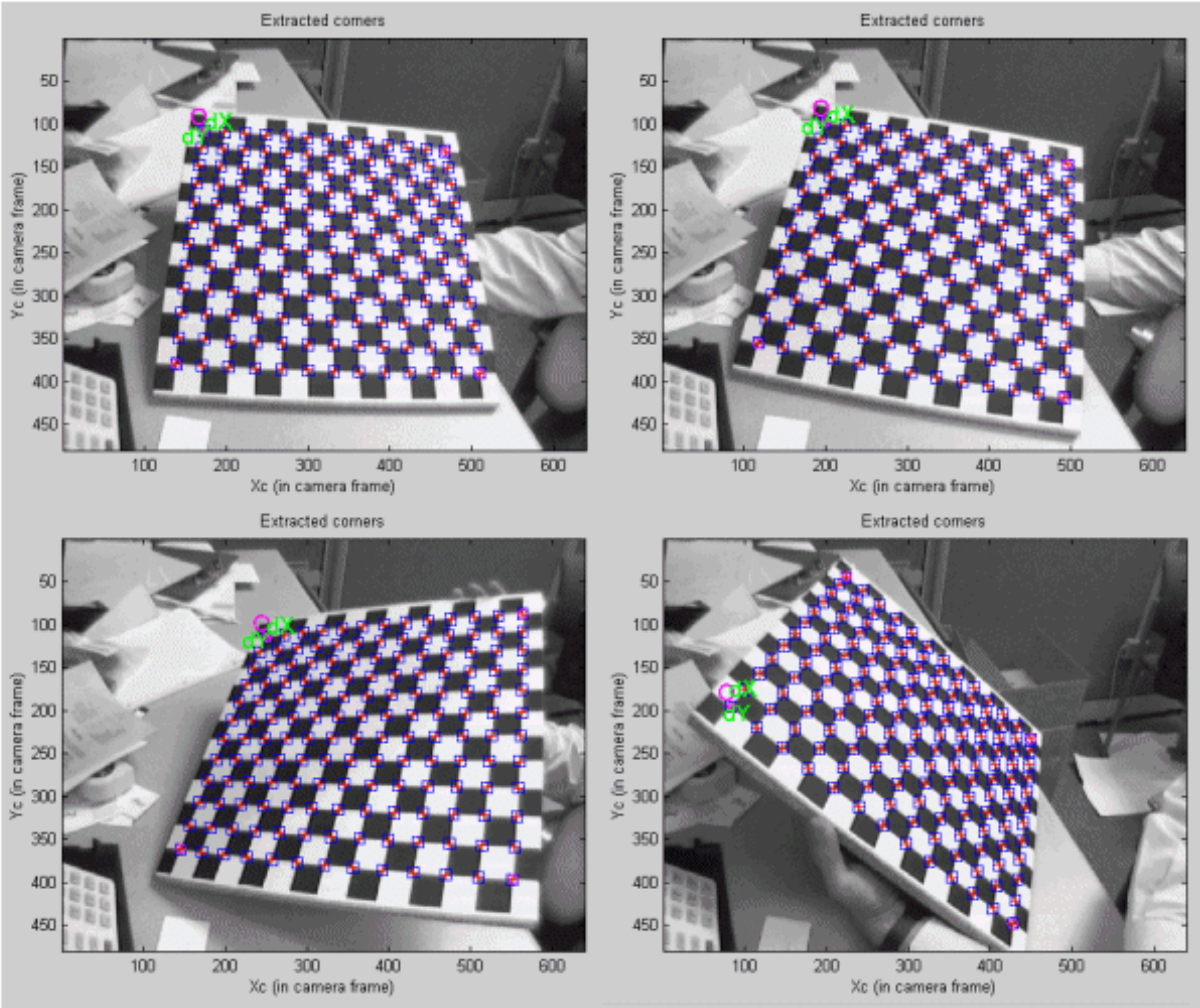


Slide credit: Yung-Yu Chuang

Step 3: corner extraction

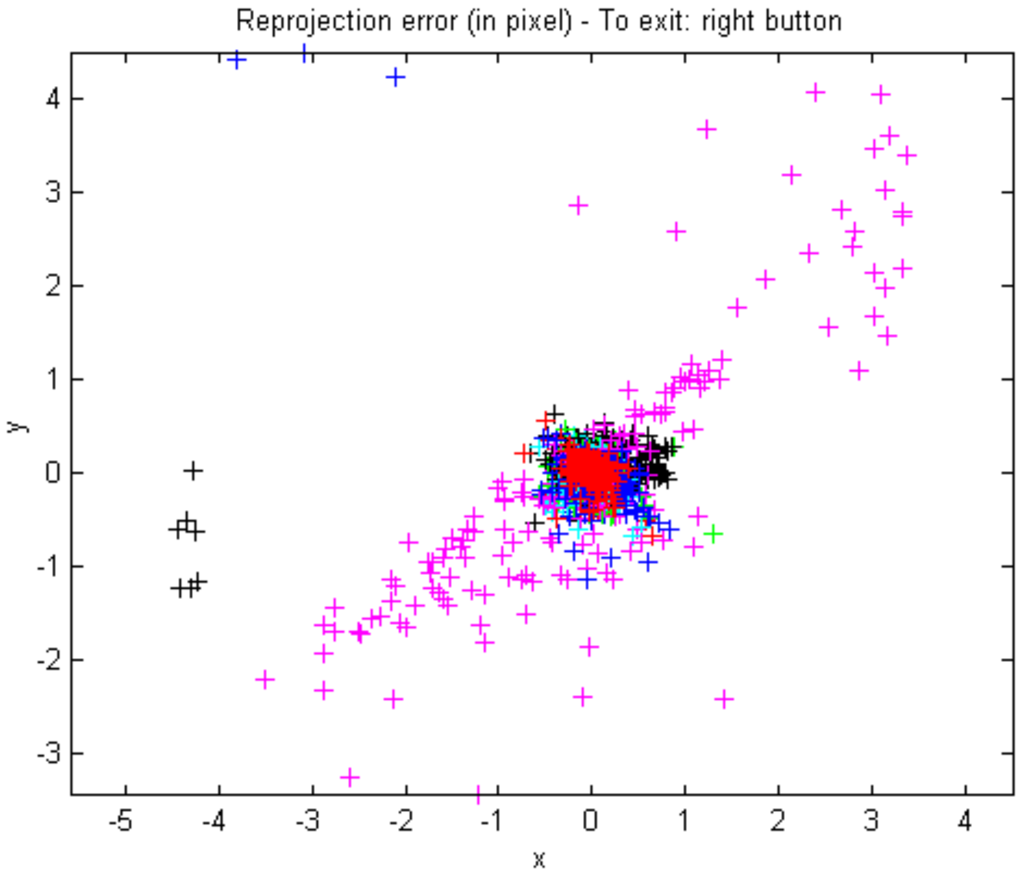


Step 3: corner extraction



Slide credit: Yung-Yu Chuang

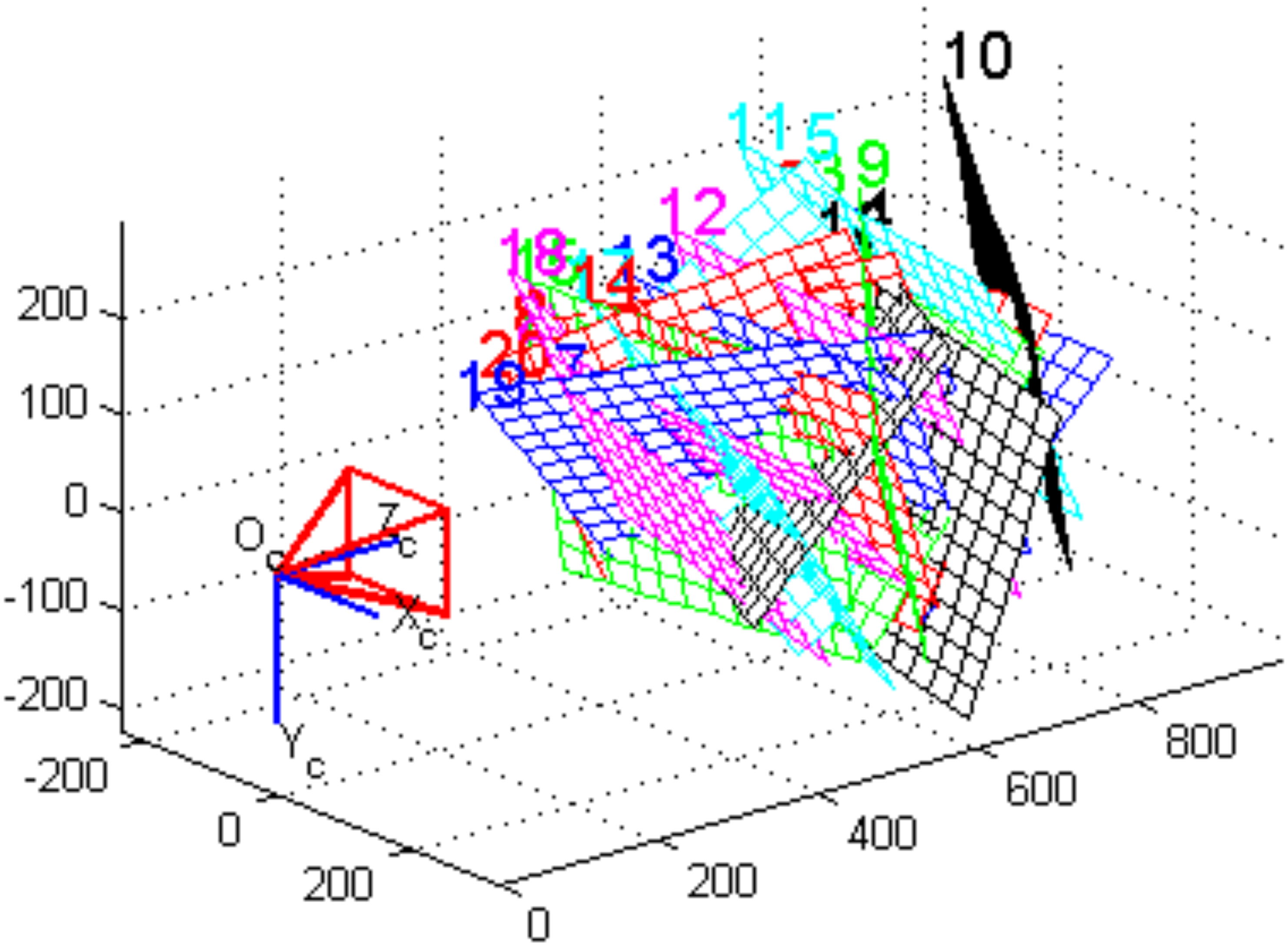
Step 4: minimize projection error




```
Calibration res

Focal Length:          fc = [ 657.46290   657.94673 ] ± [ 0.31819   0.34046 ]
Principal point:       cc = [ 303.13665   242.56935 ] ± [ 0.64682   0.59218 ]
Skew:             alpha_c = [ 0.00000 ] ± [ 0.00000  ]   => angle of pixel axes =
Distortion:            kc = [ -0.25403   0.12143   -0.00021   0.00002  0.00000 ]
Pixel error:          err = [ 0.11689   0.11500 ]
```

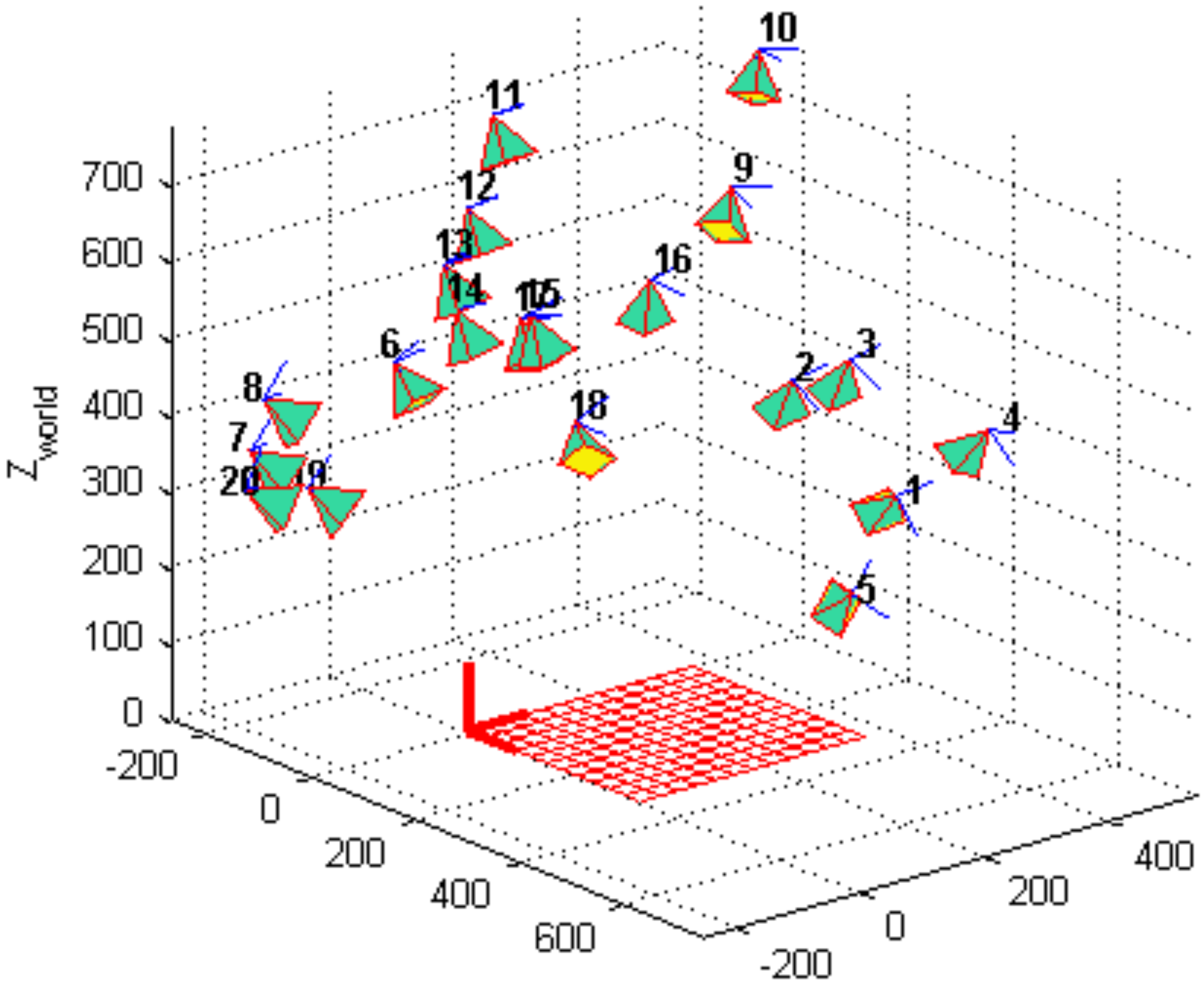
Slide credit: Yung-Yu Chuang

Step 4: camera calibration



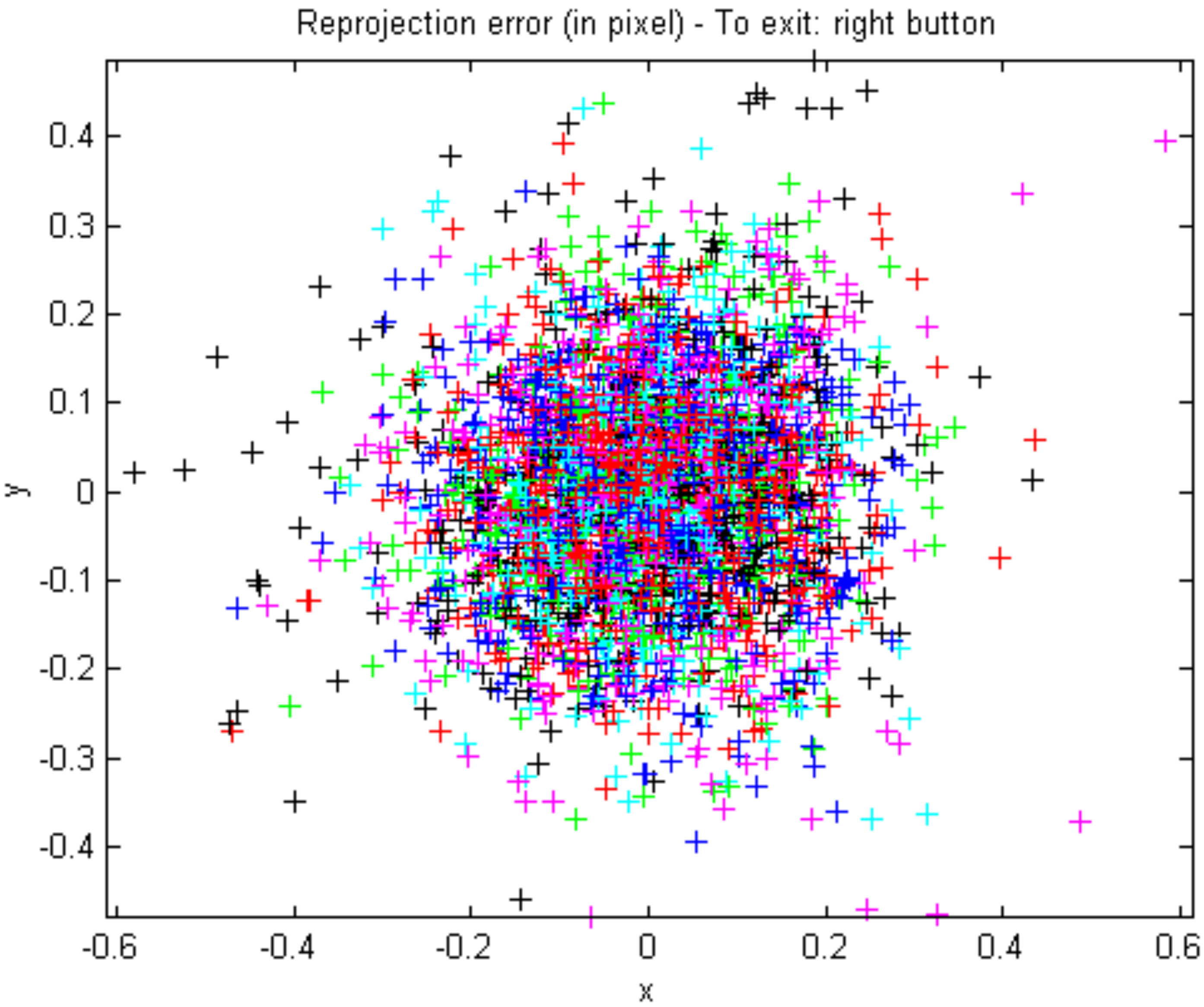
Slide credit: Yung-Yu Chuang

Step 4: camera calibration



Slide credit: Yung-Yu Chuang

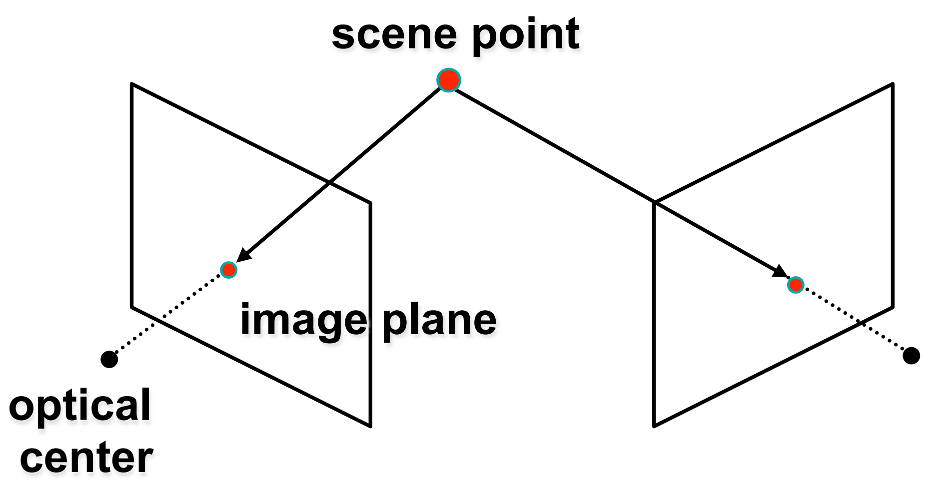
Step 5: refinement



Optimized parameters

```
Aspect ratio optimized (est_aspect_ratio = 1) -> both components of fc are estimated (DEFAULT).
Principal point optimized (center_optim=1) - (DEFAULT). To reject principal point, set center_optim=0
Skew not optimized (est_alpha=0) - (DEFAULT)
Distortion not fully estimated (defined by the variable est_dist):
     Sixth order distortion not estimated (est_dist(5)=0) - (DEFAULT) .

Main calibration optimization procedure - Number of images: 20
Gradient descent iterations: 1...2...3...4...5...done
Estimation of uncertainties...done


Calibration results after optimization (with uncertainties):

Focal Length:          fc = [ 657.46290   657.94673 ] ± [ 0.31819   0.34046 ]
Principal point:       cc = [ 303.13665   242.56935 ] ± [ 0.64682   0.59218 ]
Skew:             alpha_c = [ 0.00000 ] ± [ 0.00000  ]   => angle of pixel axes = 90.00000 ± 0.00000 degrees
Distortion:            kc = [ -0.25403   0.12143   -0.00021   0.00002  0.00000 ] ± [ 0.00248   0.00986   0.0
Pixel error:          err = [ 0.11689   0.11500 ]

Note: The numerical errors are approximately three times the standard deviations (for reference).
```

Camera parameters



Extrinsic parameters:
Camera frame 1 ←→ Camera frame 2

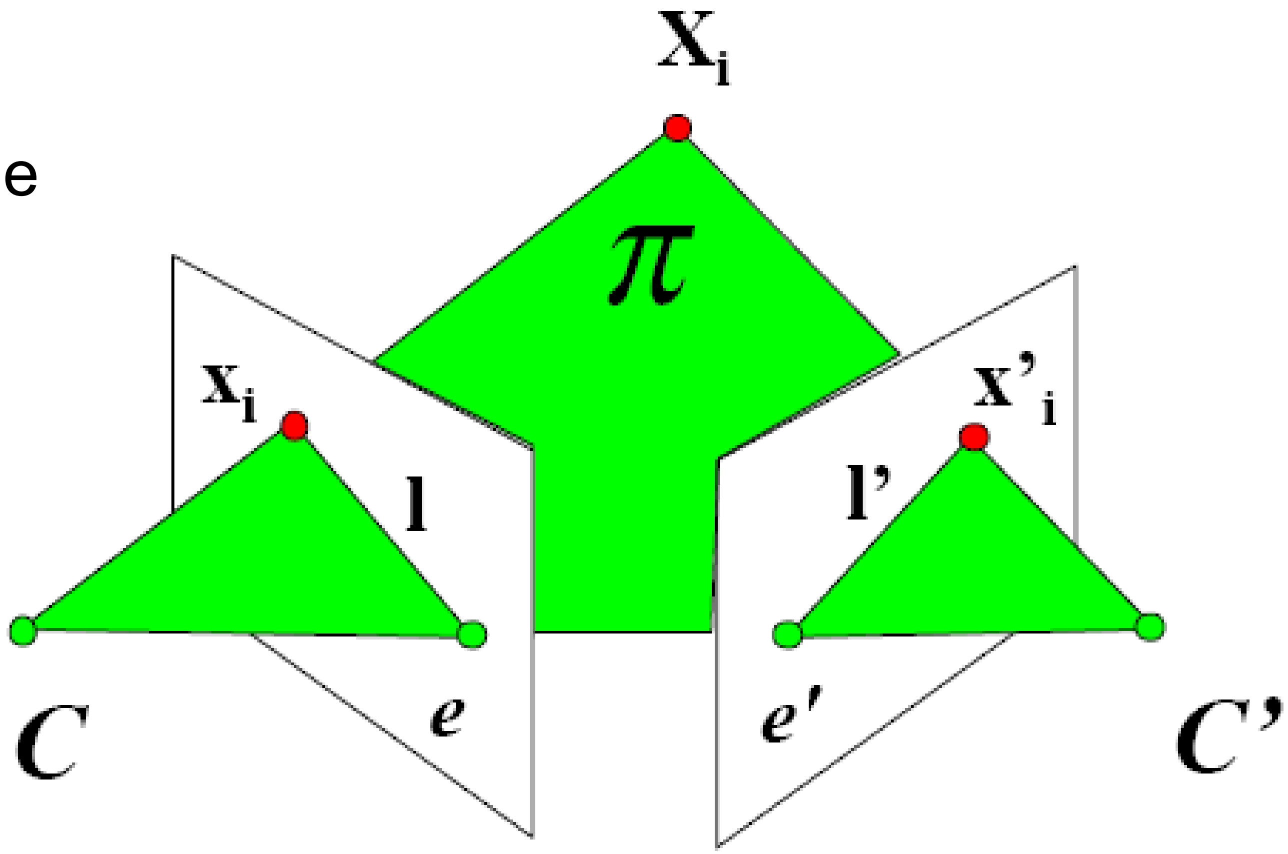
Intrinsic parameters:
Image coordinates relative to camera ←→ Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

We'll assume for now that these parameters are given and fixed.

Epipolar Geometry

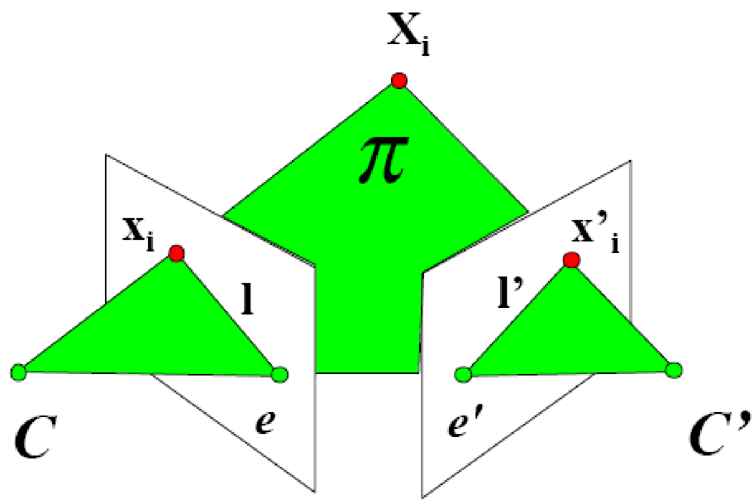
CC’: baseline
e, e’: epipole
x_ie, x’$_i$e: epipolar line
π: epipolar plane



The corresponding point must lie on the epipolar line

Epipolar Geometry

F: fundamental matrix
E: essential matrix



$$\tilde{X}'F\tilde{X} = 0$$

$$F = A'^{-T} E A^{-1}$$

$$E = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & T_x \\ -T_y & T_x & 0 \end{bmatrix} R$$