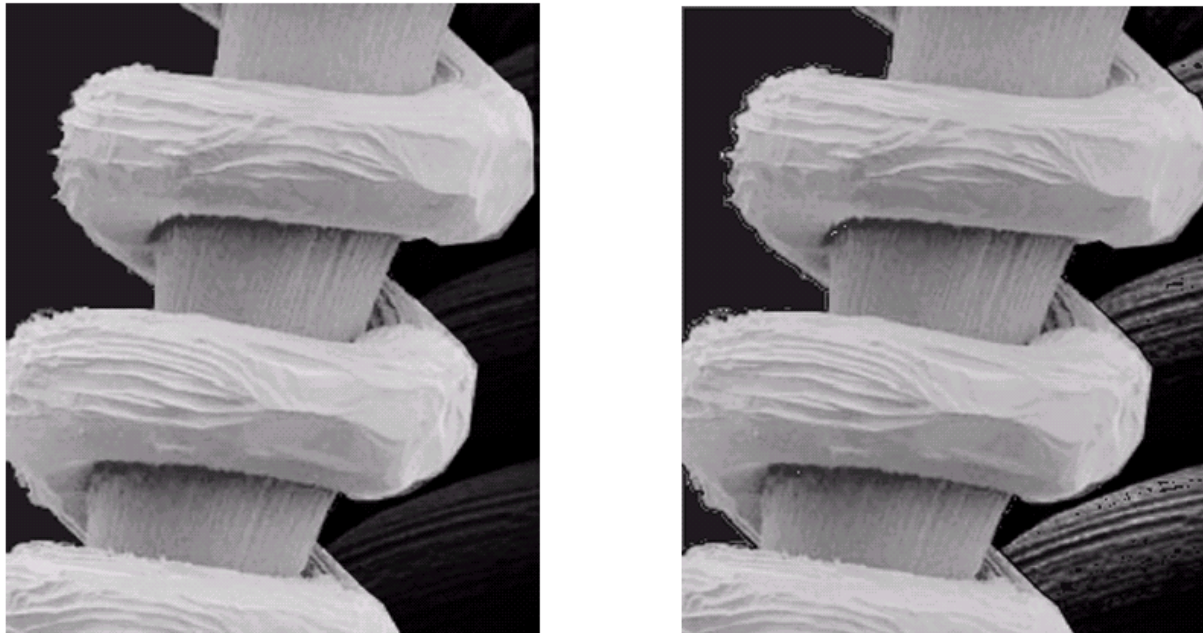


# Spatial Domain Image Processing

# Image Enhancement

**Image Enhancement** means improvement of images to be suitable for specific applications.

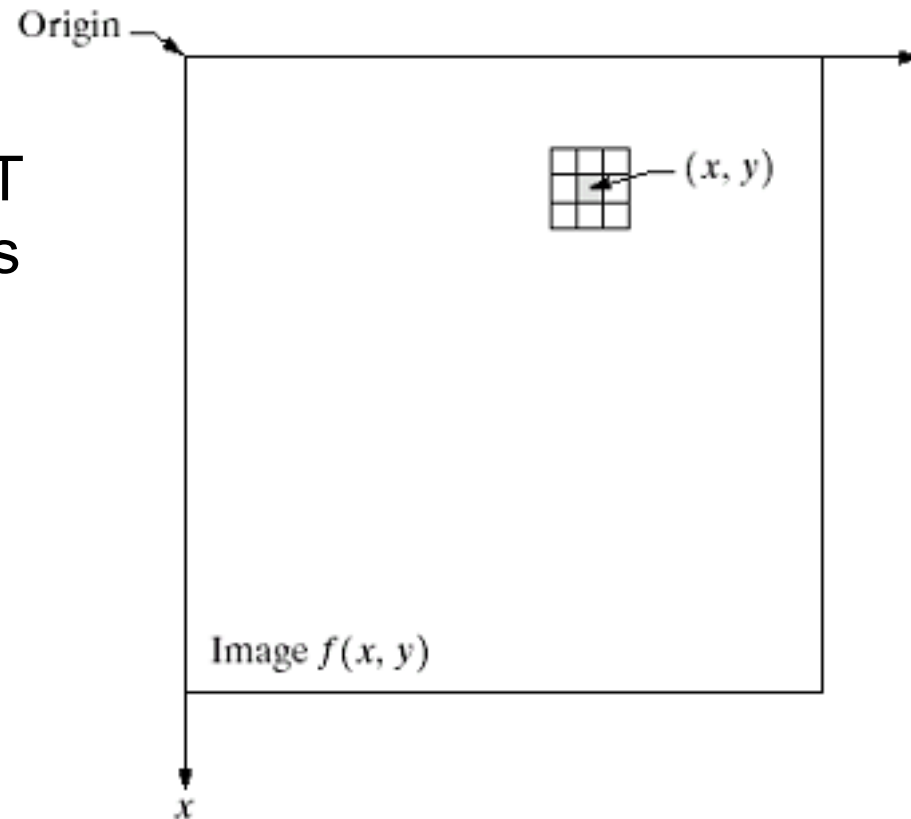
Example:



Note: each image enhancement technique that is suitable for one application may not be suitable for other applications.

# Neighbourhood

- For example, an operator  $T$  utilizes only the pixels in the area of the image spanned by the neighborhood, e.g., a  $3 \times 3$  neighborhood.

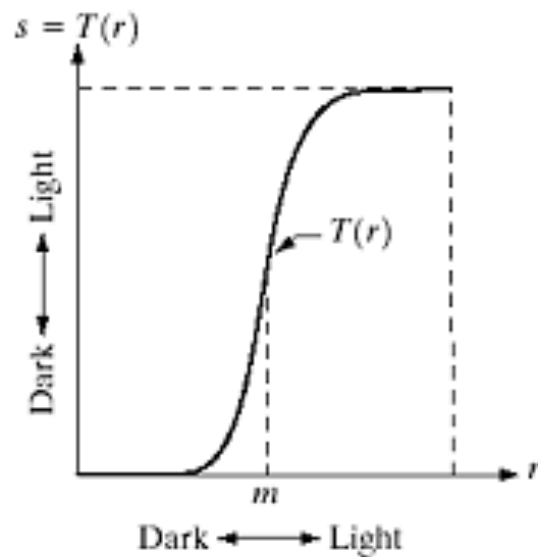


# Point processing - 1x1 neighborhood

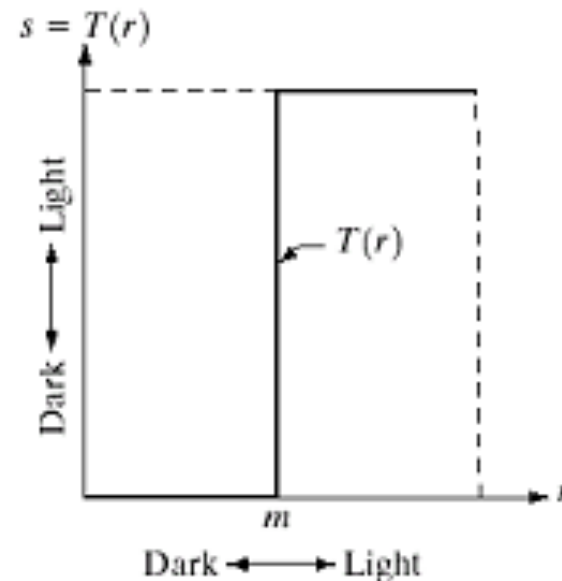
- Gray-level Transformation Function

$$g(x, y) = T[f(x, y)] \longrightarrow s = T(r)$$

Contrast Stretching



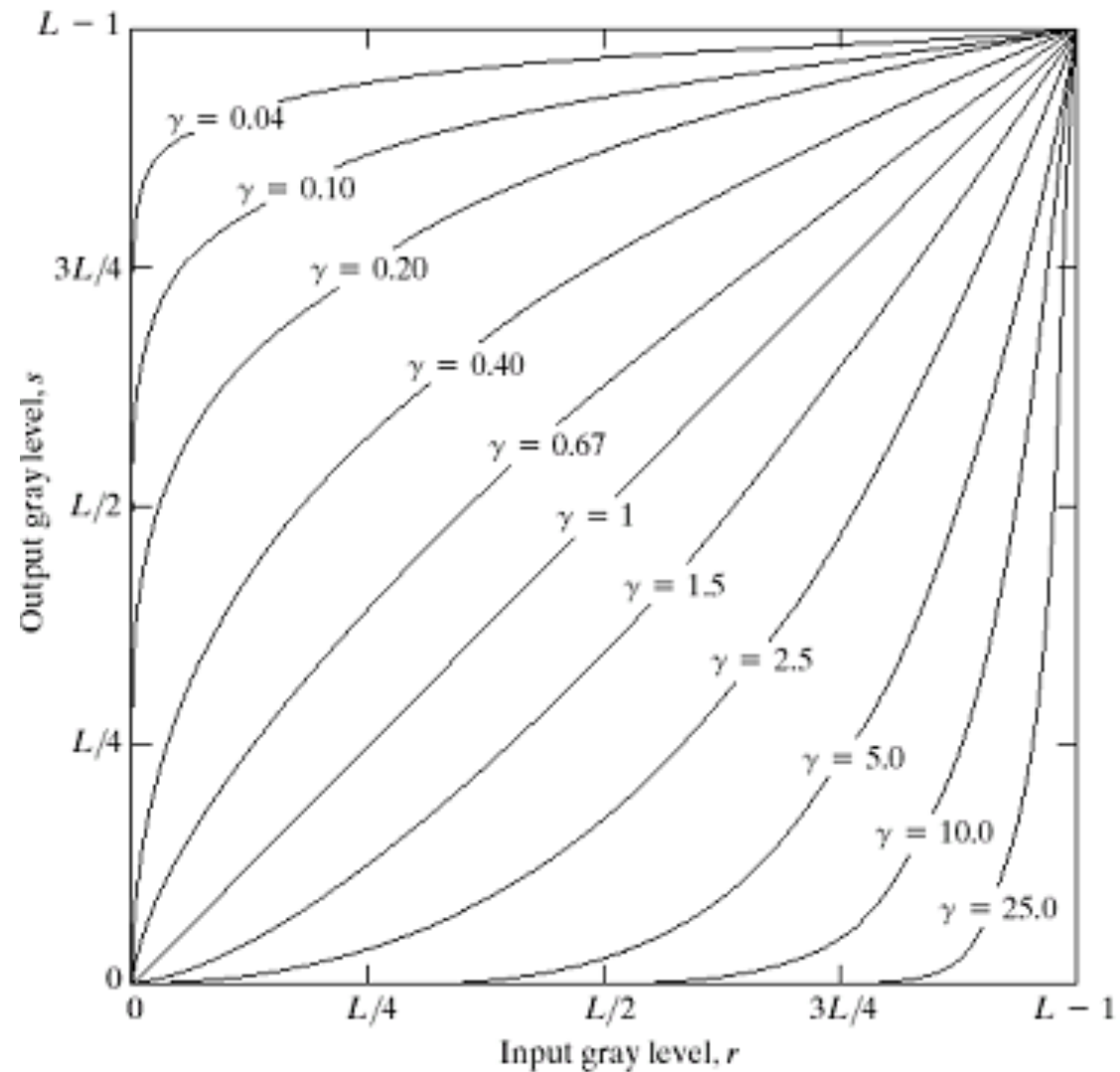
Thresholding



# Gamma Correction

$$s = c r^\gamma$$

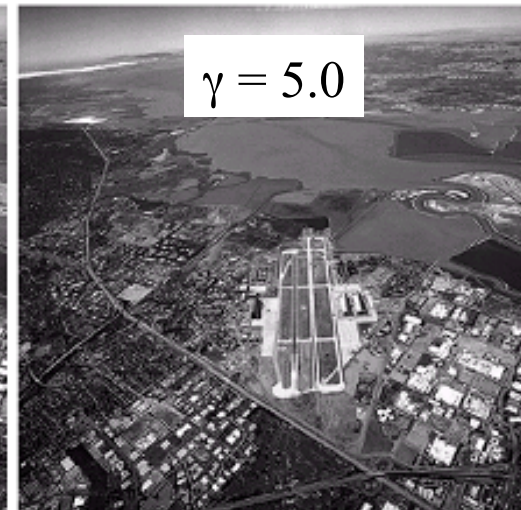
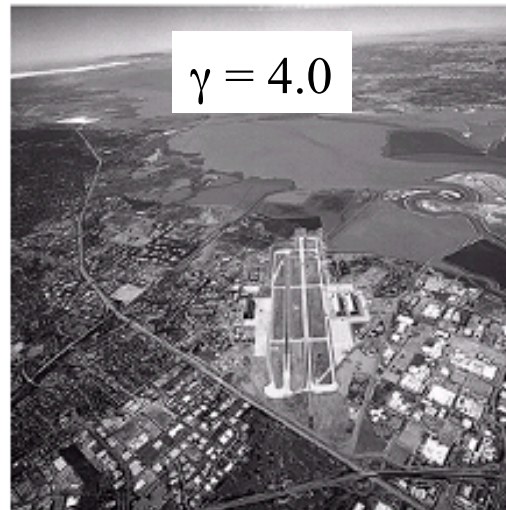
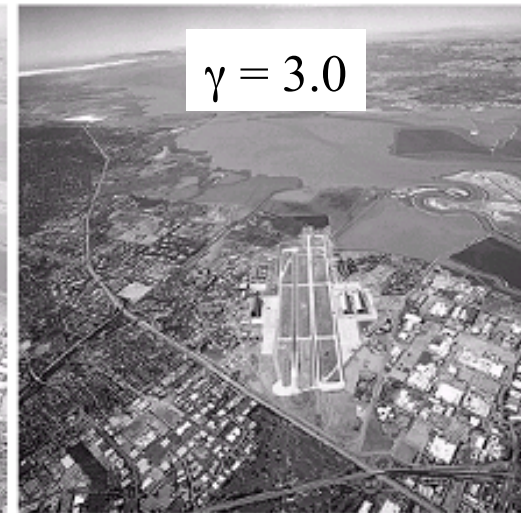
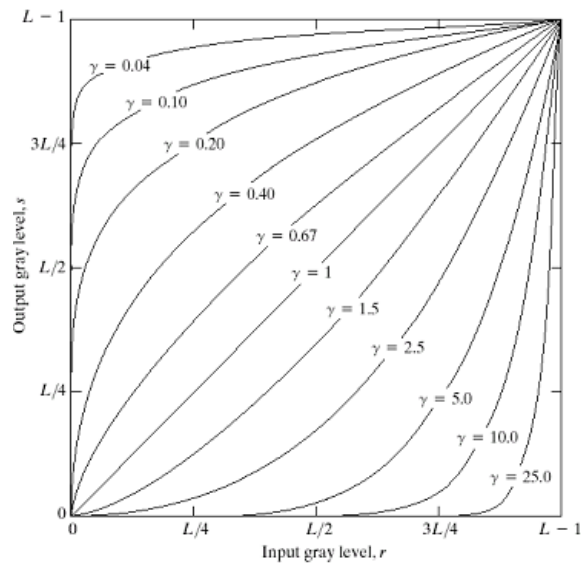
*gamma*



# Gamma Correction

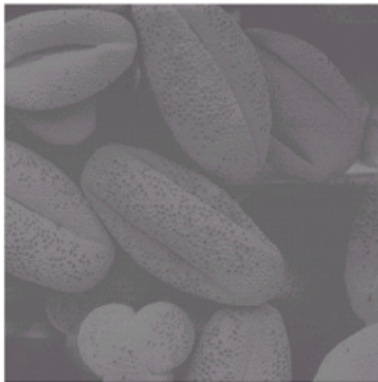
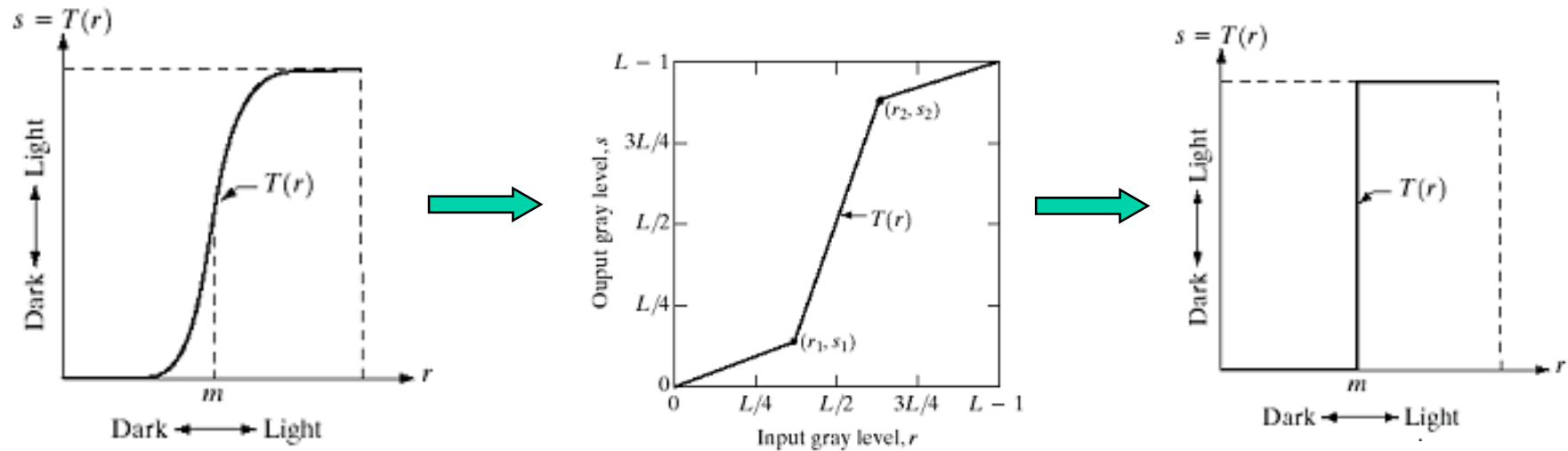
$$s = c r^\gamma$$

*gamma*

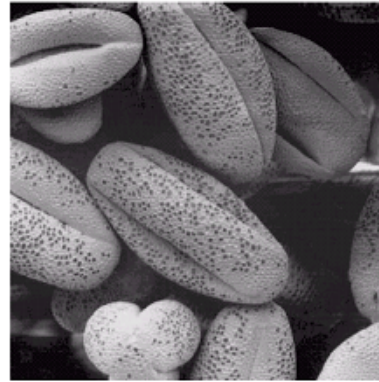




# Contrast Stretching



Full-Scale  
Histogram Stretch



$$(r_1, s_1) = (r_{\min}, 0)$$

$$(r_2, s_2) = (r_{\max}, L-1)$$



Thresholding

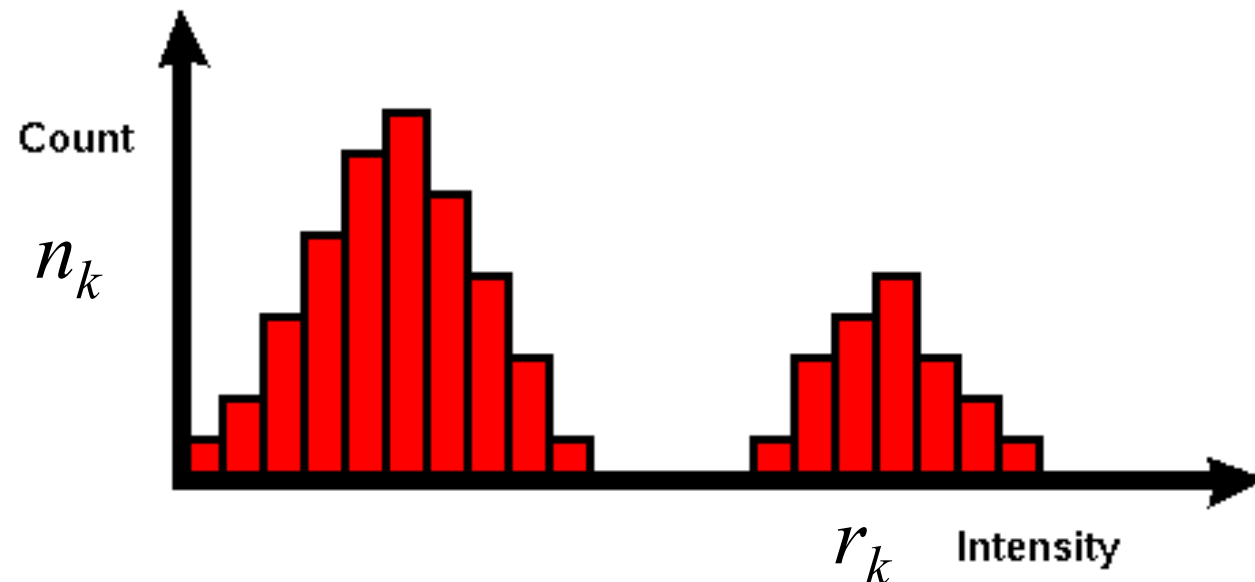
# Histogram

- Histogram of a digital image is a distribution function

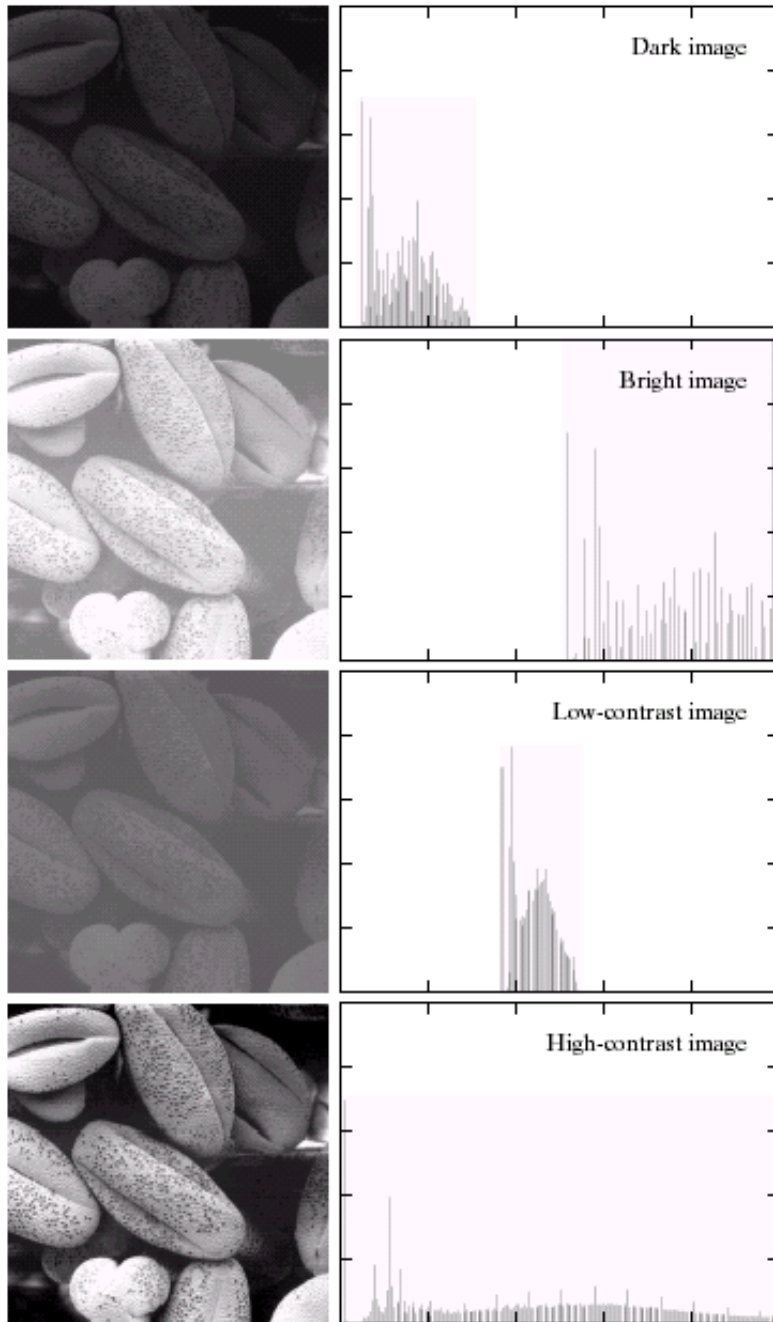
$$h(r_k) = n_k$$

– where  $r_k$  is the  $k$ th gray level

and  $n_k$  is the number of pixels having gray level  $r_k$







## Different types of histograms:

- Normalized histogram

$$p(r_k) = n_k / n, \quad k = 1, \dots, L$$

- Histogram is useful for
  - image enhancement
  - image compression
  - image segmentation
  - etc.

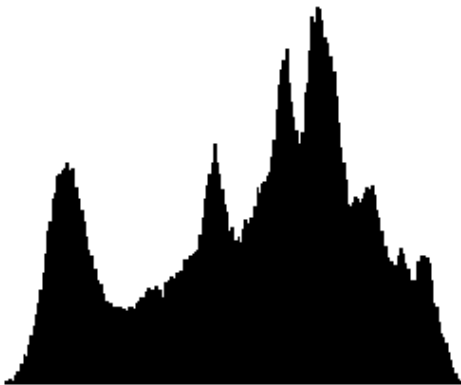
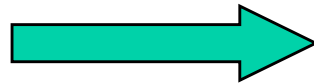
→ Want to have a more flat histogram !

**=> Histogram Equalization**

# Histogram Equalization



Image  
Enhancement



Histogram  
Equalization



To make histogram distributed uniformly

# Algorithm of Histogram Equalization

1. Compute the histogram of the input image:

$$h(k) = \#\{(x,y) | f(x,y)=k\}, \text{ where } k = 0 \text{ to } 255.$$

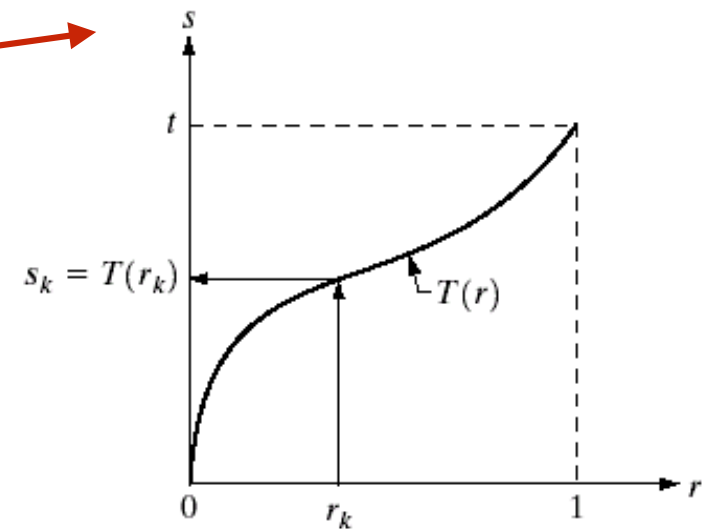
2. Compute the transformation function:

$$T(k) = 255 * \sum_{j=0}^k \frac{h(j)}{n}$$

Cumulative normalized histogram

3. Transform the value of each pixel by

$$g(x,y) = T(f(x,y))$$



# Histogram Equalization Example

Intensity	# pixels
0	20
1	5
2	25
3	10
4	15
5	5
6	10
7	10
Total	100

Accumulative Sum of $P_r$
$20/100 = 0.2$
$(20+5)/100 = 0.25$
$(20+5+25)/100 = 0.5$
$(20+5+25+10)/100 = 0.6$
$(20+5+25+10+15)/100 = 0.75$
$(20+5+25+10+15+5)/100 = 0.8$
$(20+5+25+10+15+5+10)/100 = 0.9$
$(20+5+25+10+15+5+10+10)/100 = 1.0$
1.0

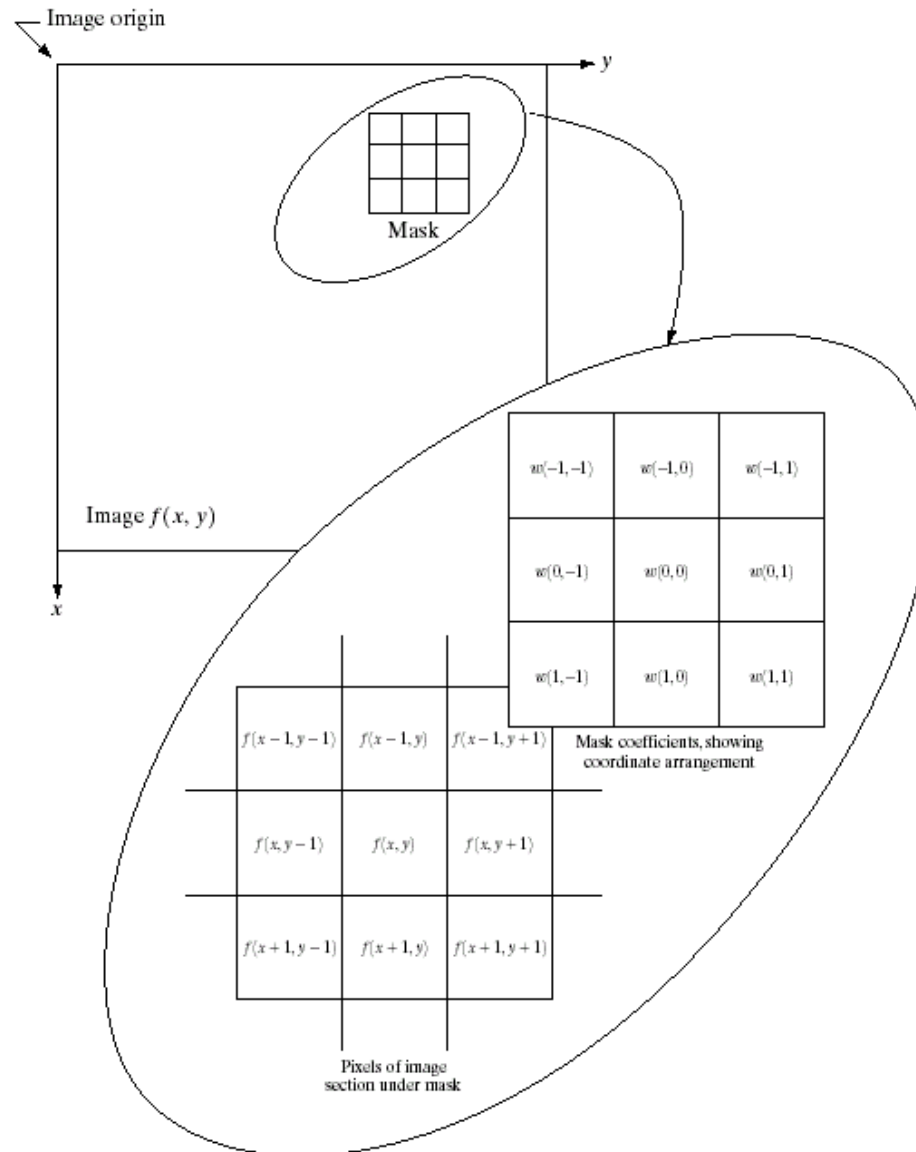
# Histogram Equalization Example

Intensity (r)	No. of Pixels (n <sub>j</sub> )	Acc Sum of P <sub>r</sub>	Output value	Quantized Output (s)
0	20	0.2	$0.2 \times 7 = 1.4$	1
1	5	0.25	$0.25 * 7 = 1.75$	1
2	25	0.5	$0.5 * 7 = 3.5$	3
3	10	0.6	$0.6 * 7 = 4.2$	4
4	15	0.75	$0.75 * 7 = 5.25$	5
5	5	0.8	$0.8 * 7 = 5.6$	5
6	10	0.9	$0.9 * 7 = 6.3$	6
7	10	1.0	$1.0 \times 7 = 7$	7
Total	100			

# Mask Processing

# Mask processing - Spatial Filtering

- [Filter](#), Mask, Kernel, Template, Window
- Coefficients
- [Linear Filtering](#) vs [Nonlinear Filtering](#)  
(e.g., median filtering)



**FIGURE 3.32** The mechanics of spatial filtering. The magnified drawing shows a  $3 \times 3$  mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.



# Linear Smoothing Filters

– averaging filters

 $\frac{1}{9} \times$ 

1	1	1
1	1	1
1	1	1

Box Filter

 $\frac{1}{16} \times$ 

1	2	1
2	4	2
1	2	1

Weighted Average

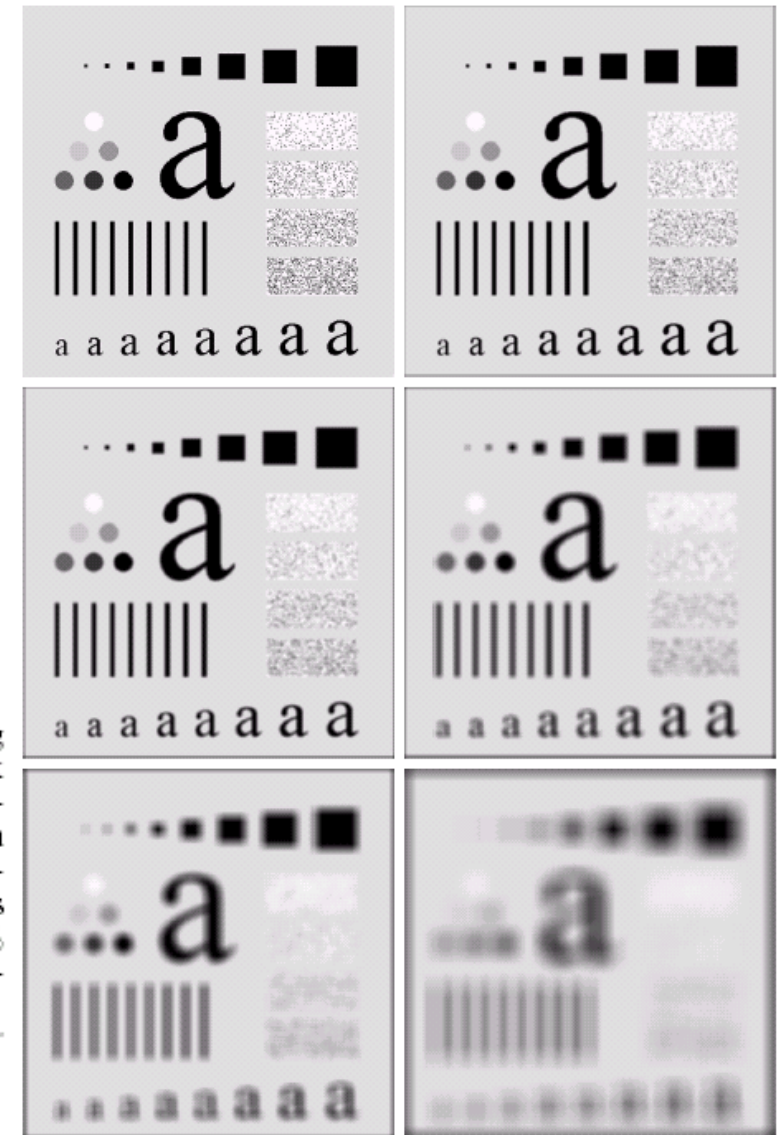
$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}.$$

# Linear Smoothing Filters

## – averaging filters

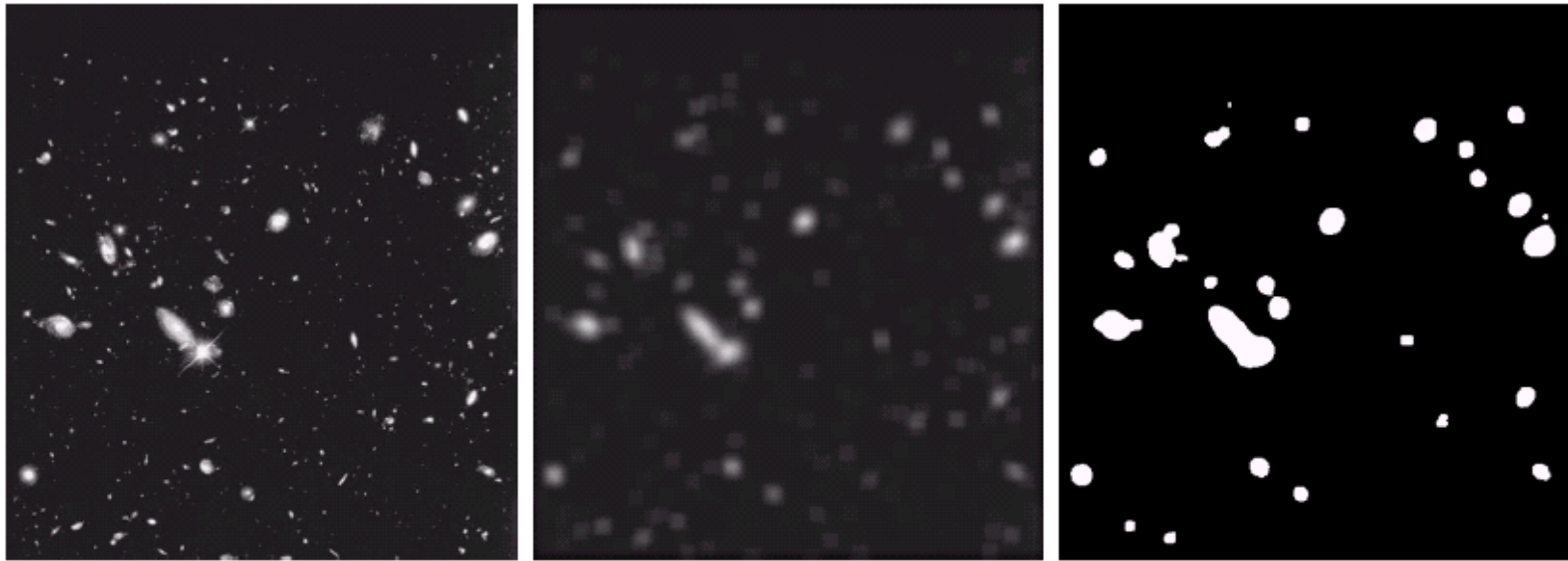
a b  
c d  
e f

**FIGURE 3.35** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $n = 3, 5, 9, 15$ , and  $35$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45$ , and  $55$  pixels, respectively; their borders are  $25$  pixels apart. The letters at the bottom range in size from  $10$  to  $24$  points, in increments of  $2$  points; the large letter at the top is  $60$  points. The vertical bars are  $5$  pixels wide and  $100$  pixels high; their separation is  $20$  pixels. The diameter of the circles is  $25$  pixels, and their borders are  $15$  pixels apart; their gray levels range from  $0\%$  to  $100\%$  black in increments of  $20\%$ . The background of the image is  $10\%$  black. The noisy rectangles are of size  $50 \times 120$  pixels.



# Linear Smoothing Filters

## – averaging filters



a b c

**FIGURE 3.36** (a) Image from the Hubble Space Telescope. (b) Image processed by a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Application: Averaging before Thresholding

# Nonlinear Smoothing Filters

## – Order-Statistics Filters

### Median Filter

- the 50<sup>th</sup> percentile of a ranked set of numbers
- effective for reducing impulse noise,  
or salt-and-pepper noise

### Max Filter

- the 100<sup>th</sup> percentile filter

### Min Filter

- the 0<sup>th</sup> percentile filter

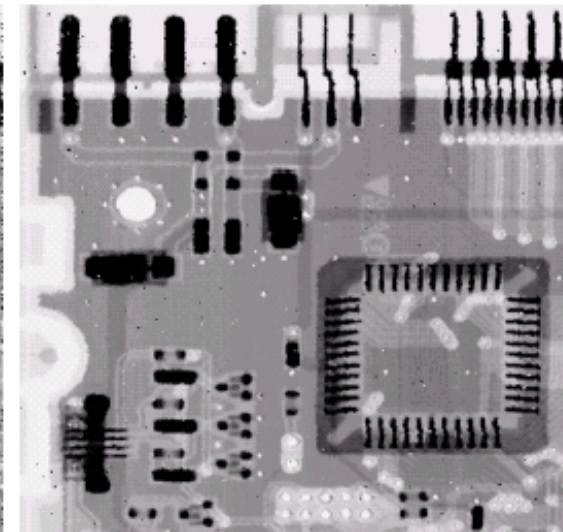
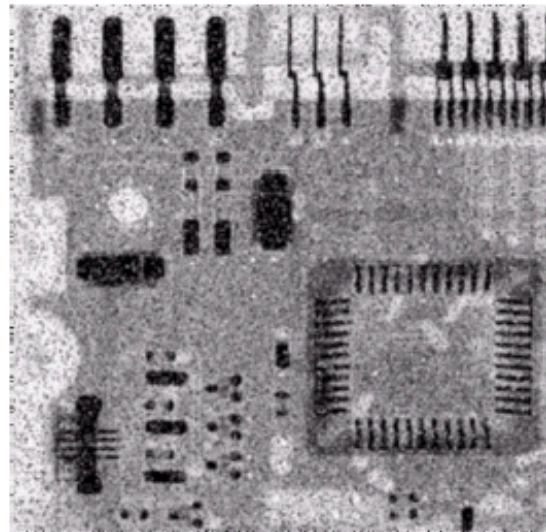
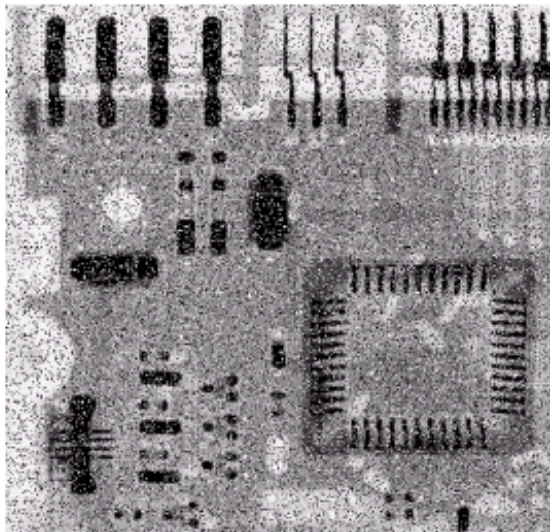
# Nonlinear Smoothing Filters

## – Order-Statistics Filters

Salt-and-pepper  
noise image

3x3 averaging filter

3x3 median filter



a b c

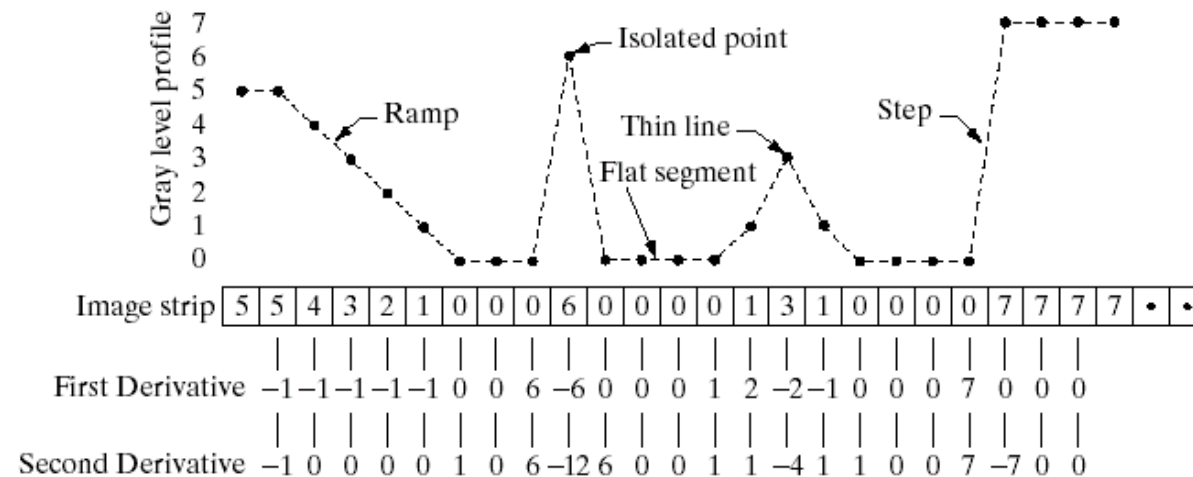
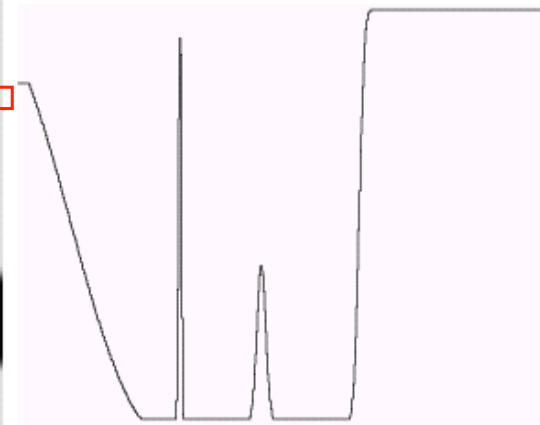
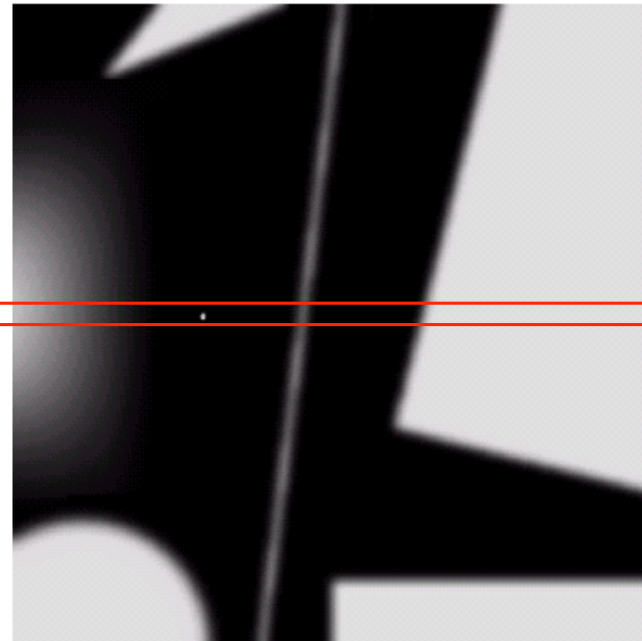
**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

# Edge detection and sharpening

a b  
c

**FIGURE 3.38**

(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.  
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).

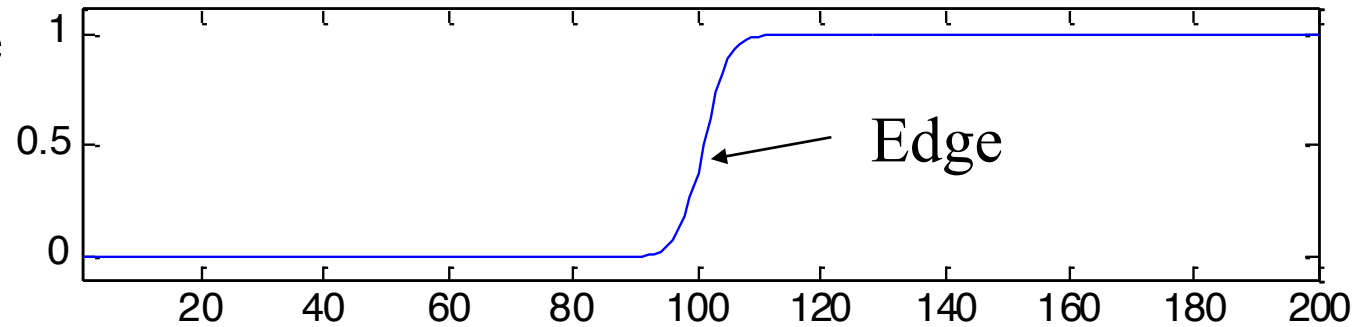
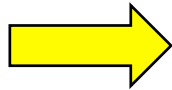


for edge detection  
for sharpening

# Edge detection

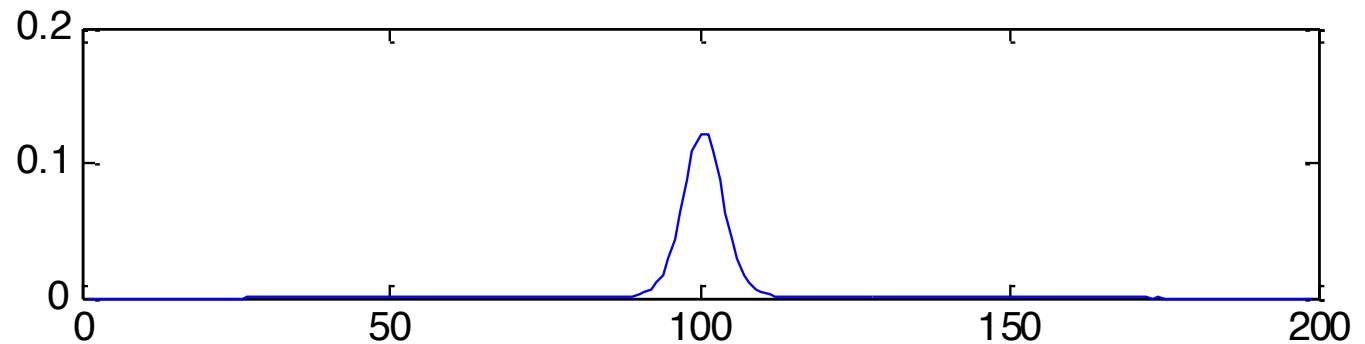
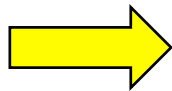
Intensity profile

$p(x)$



1<sup>st</sup> derivative

$\frac{dp}{dx}$





# Edge detection

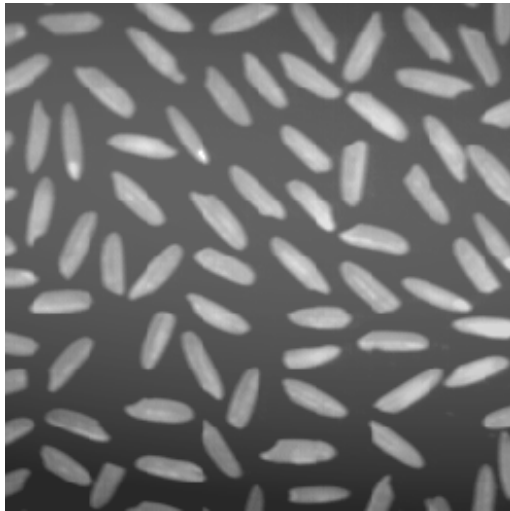
## Sobel operators

-1	0	1
-2	0	2
-1	0	1

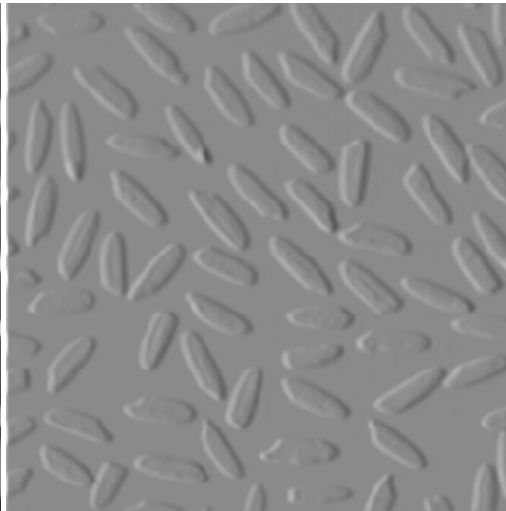
to compute  $\frac{\partial P}{\partial x}$

-1	-2	-1
0	0	0
1	2	1

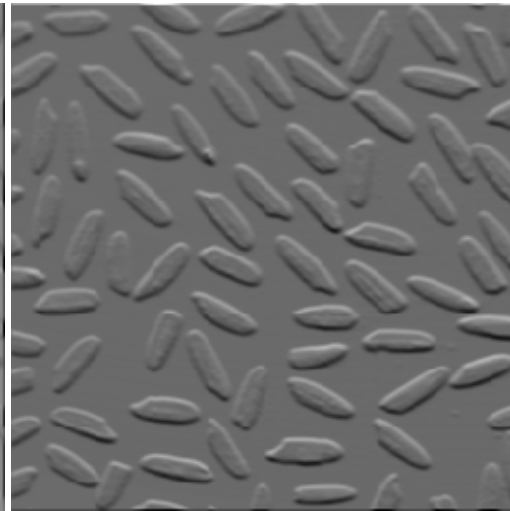
to compute  $\frac{\partial P}{\partial y}$



$P$



$\frac{\partial P}{\partial x}$

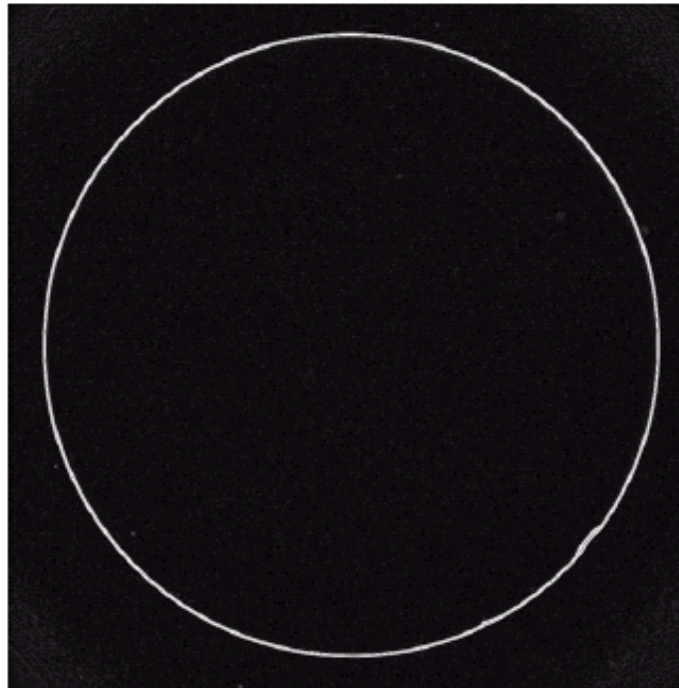
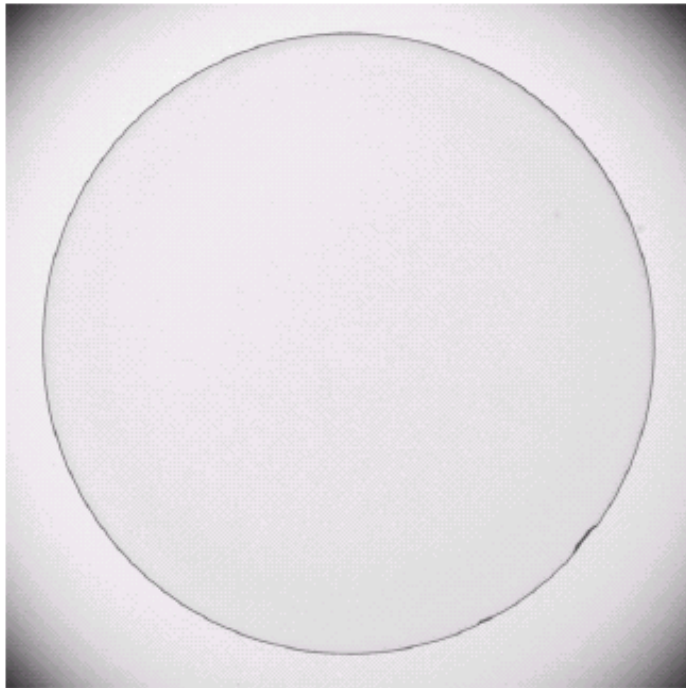


$\frac{\partial P}{\partial y}$

# Edge detection

Gradient magnitude

$$|\nabla P| = \sqrt{\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial y}\right)^2}$$



a b

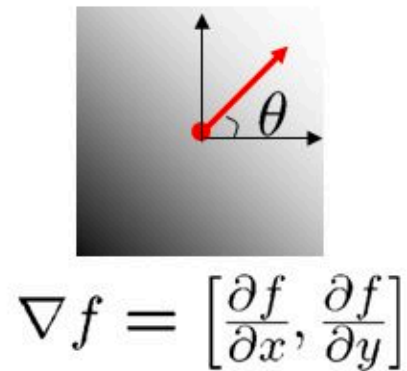
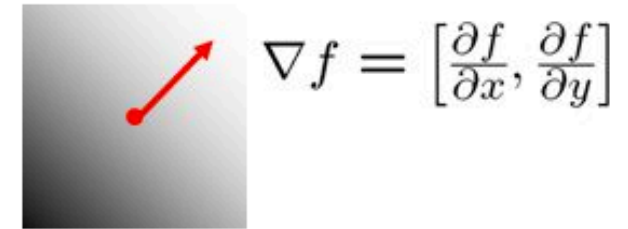
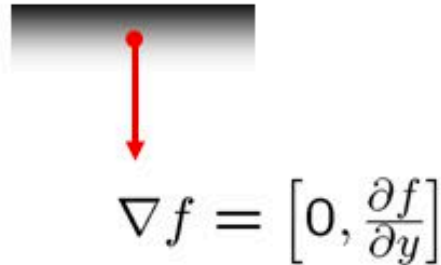
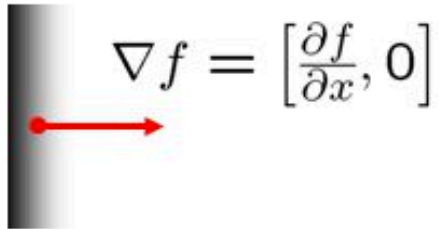
**FIGURE 3.45**

Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).

(b) Sobel gradient.

(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

# Gradient direction



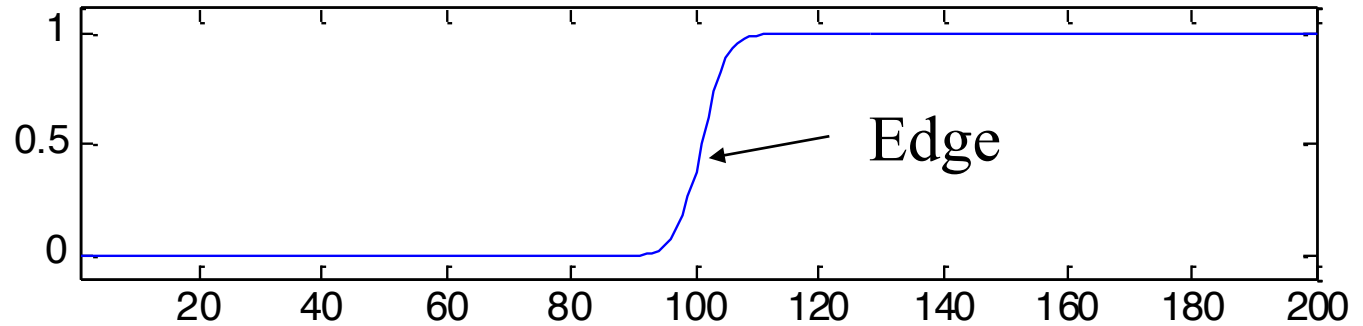
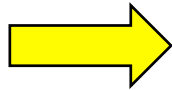
The gradient direction is given by:

$$\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

# Laplacian sharpening

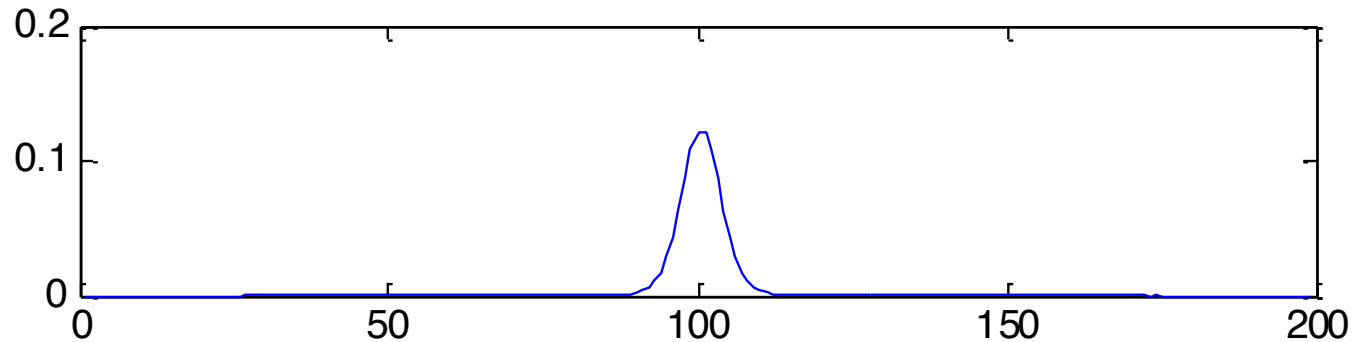
Intensity profile

$p(x)$



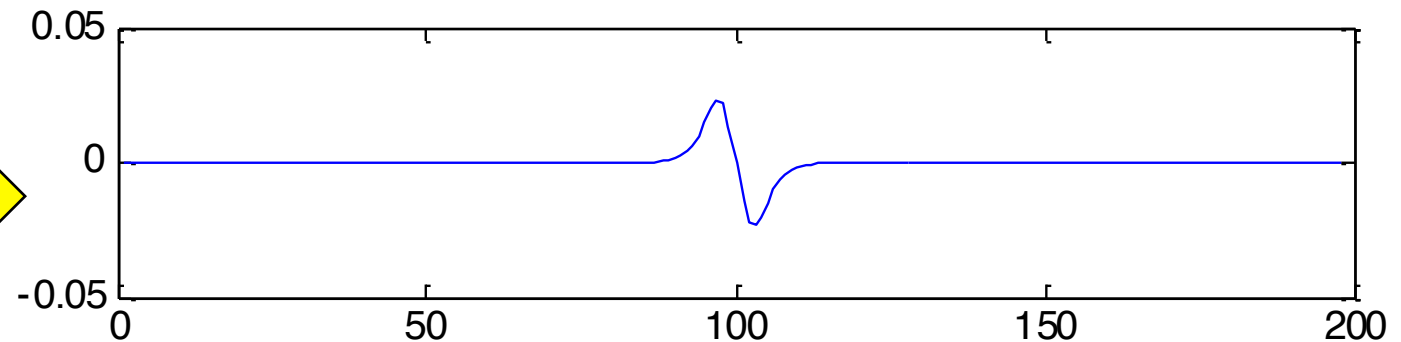
1<sup>st</sup> derivative

$\frac{dp}{dx}$

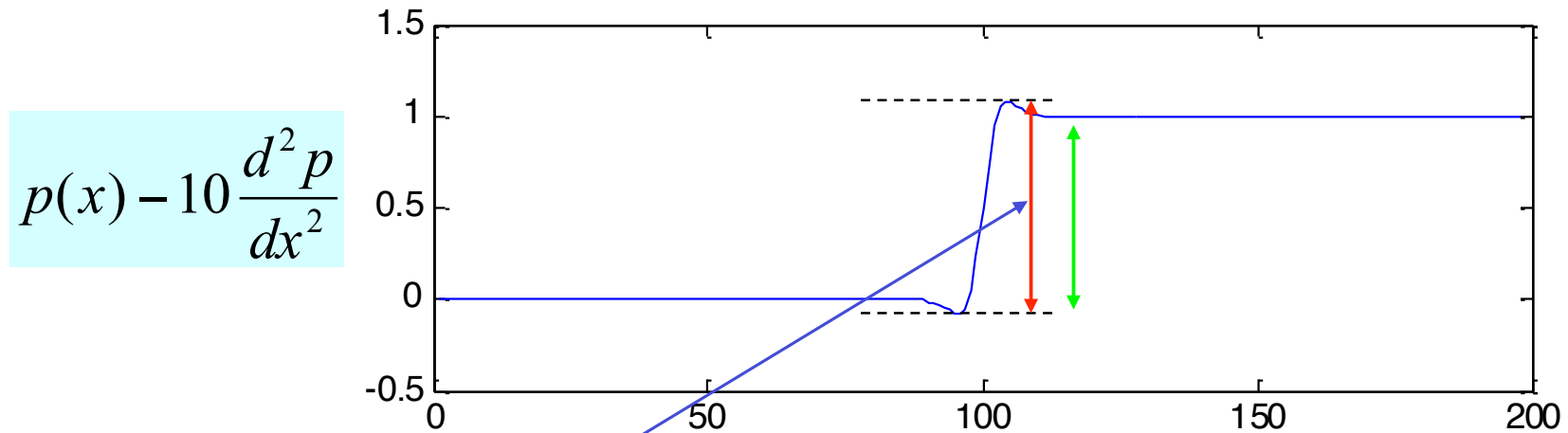
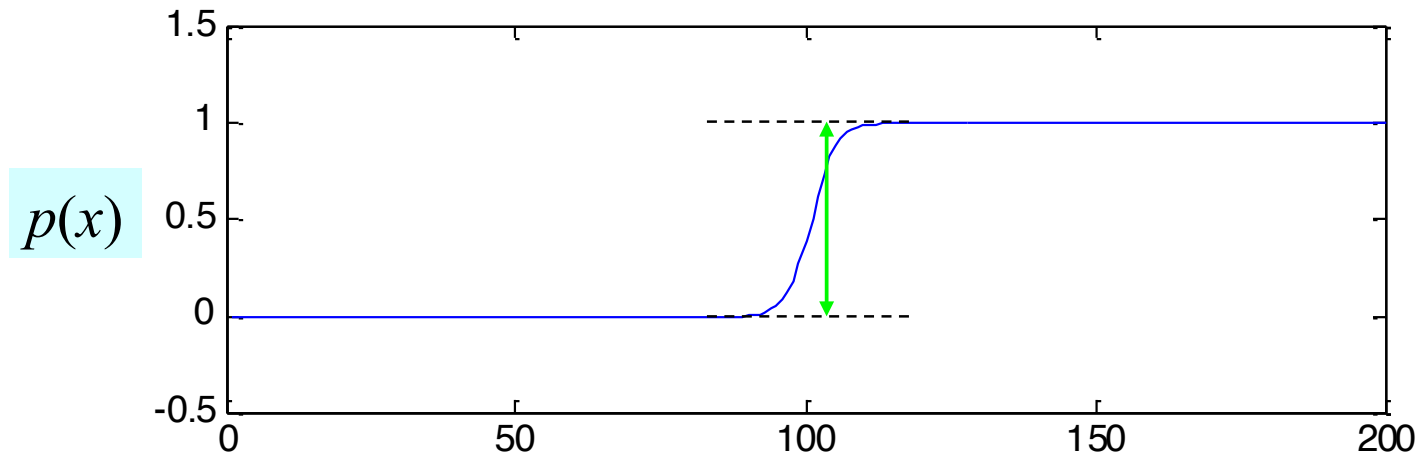


2<sup>nd</sup> derivative

$\frac{d^2 p}{dx^2}$

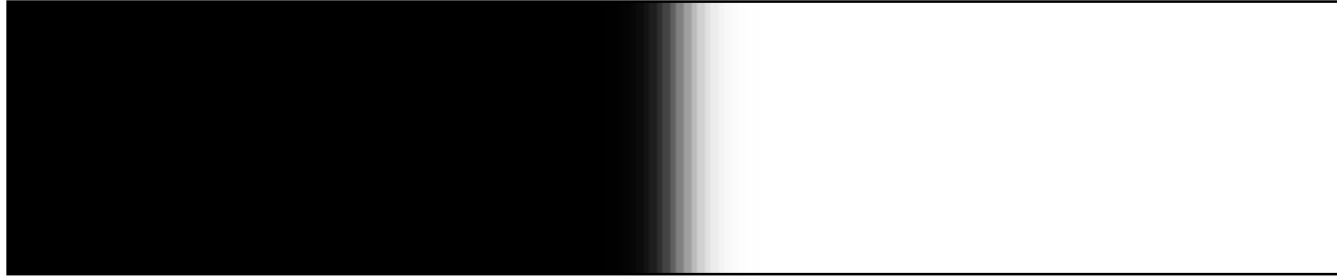


# Laplacian sharpening



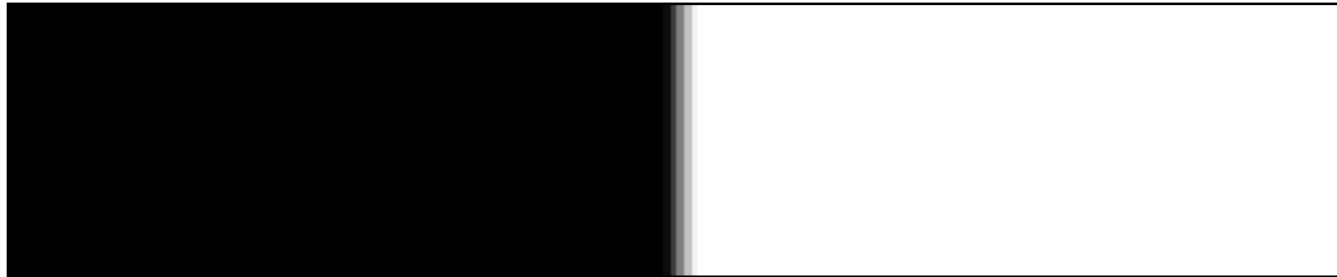
Laplacian sharpening results in larger intensity discontinuity near the edge.

# Laplacian sharpening



Before sharpening

$p(x)$



After sharpening

$p(x) - 10 \frac{d^2 p}{dx^2}$

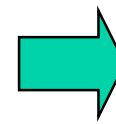
# Laplacian sharpening

Used for estimating image Laplacian

$$\nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}$$

-1	-1	-1
-1	8	-1
-1	-1	-1

0	-1	0
-1	4	-1
0	-1	0

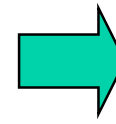


The center of the mask is positive

or

1	1	1
1	-8	1
1	1	1

0	1	0
1	-4	1
0	1	0



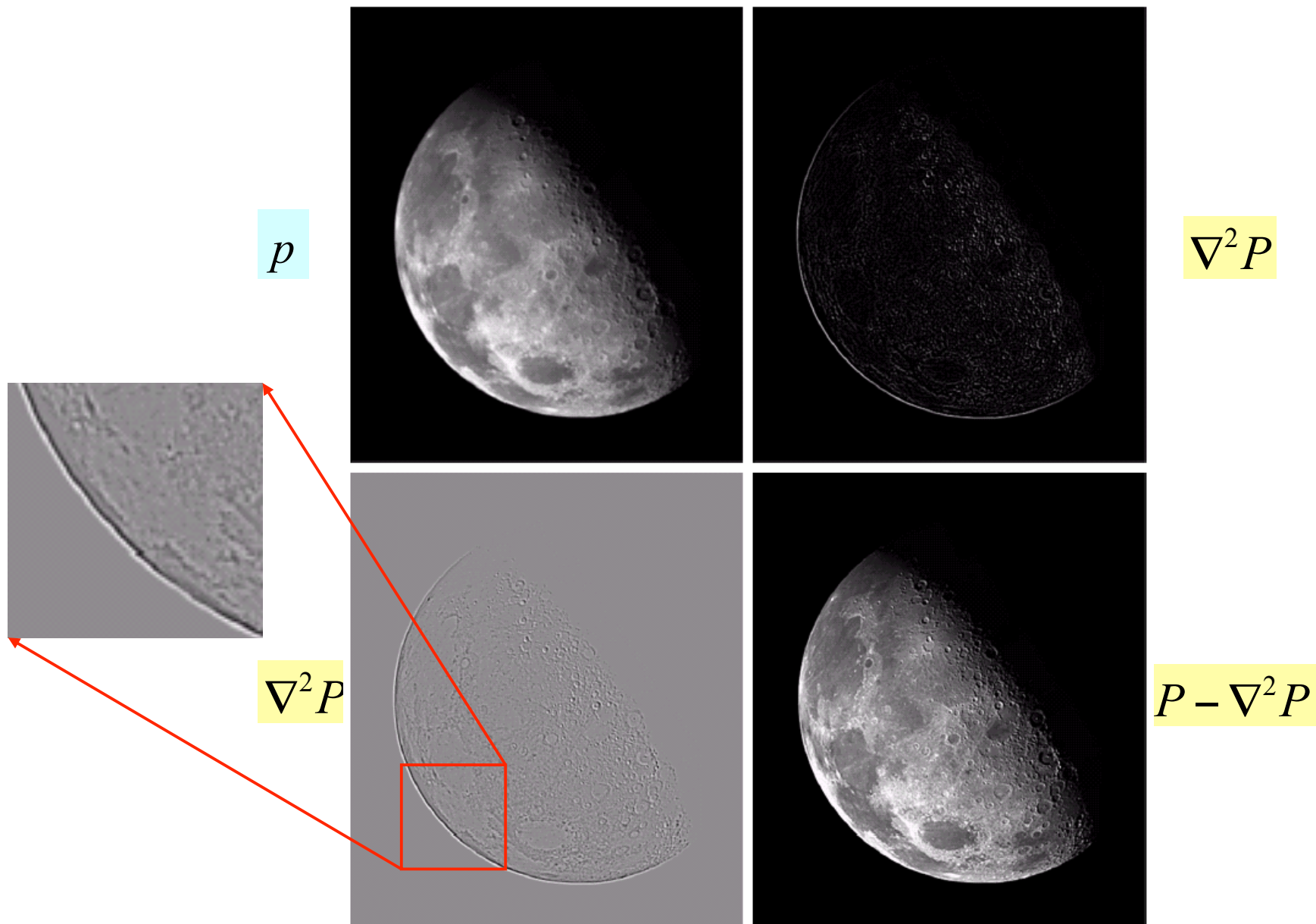
The center of the mask is negative

Application: Enhance edge, line, point

Disadvantage: Enhance noise



# Laplacian sharpening



# Laplacian sharpening

Mask for  
 $P - \nabla^2 P$

0	-1	0
-1	5	-1
0	-1	0

or

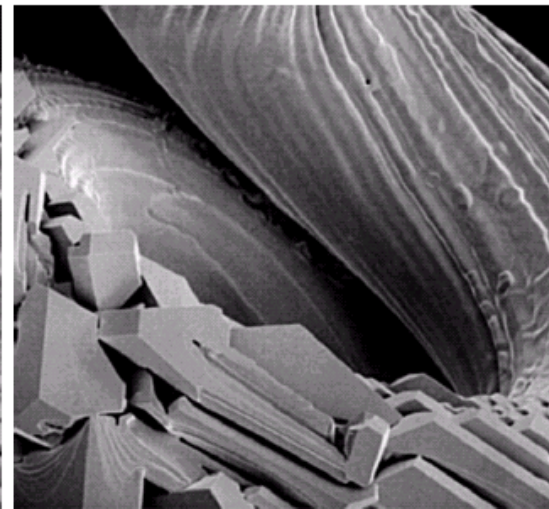
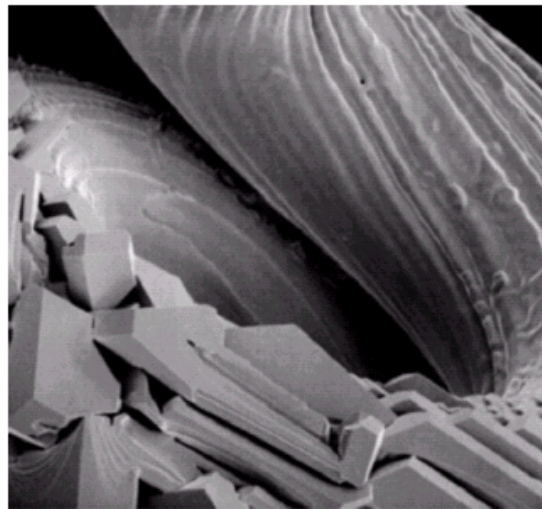
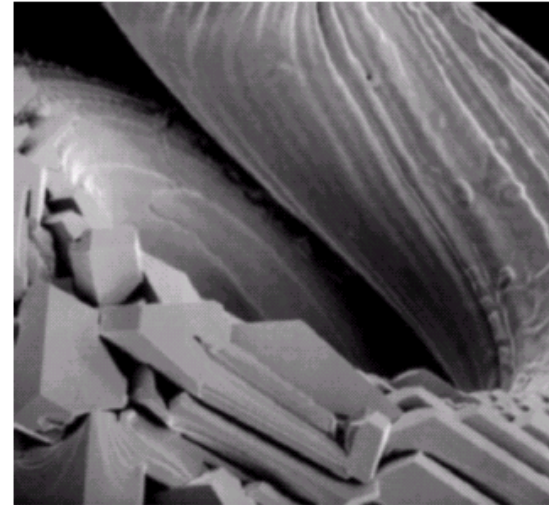
-1	-1	-1
-1	9	-1
-1	-1	-1

Mask for  
 $\nabla^2 P$

1	1	1
1	-8	1
1	1	1

or

0	1	0
1	-4	1
0	1	0



a b c  
d e

**FIGURE 3.41** (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Kansas. Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2<sup>nd</sup> Edition.

# Digital Image Processing

Kuan-Wen Chen  
2018/3/8